PART 2 STRUCTURAL ANALYSIS

STRUCTURAL ANALYSIS

CHAPTER V

Article 17 General

Structural analysis consists of the determination of the effects caused by the actions on the whole or part of the structure, with the purpose of verifying the Ultimate and Serviceability Limit States.

COMMENTS

Structural analysis provides results on an overall level (reactions, movements), on a sectional level (forces, curvatures, elongation), or on a local level (stresses, deformations), that are used for the dimensioning or verification of the various limits states.

Article 18 Idealization of the structure

18.1 Structural models

In order to carry out the analysis, both the geometry of the structure and the actions and support conditions are idealised by means of a suitable mathematical model. The chosen model must be able to reproduce the dominant structural behaviour at all times.

For the analysis, the structural elements are classified as one-dimensional when one of their dimensions is much larger than the others, two-dimensional, when one dimension is small compared with the other two; and three-dimensional when none of the dimensions is noticeably larger than the others.

COMMENTS

Supports, beams and arches are considered one-dimensional, provided that their length exceeds double the value of the total edge. Plates, shells and membranes are considered to be two-dimensional.

Other structural model classifications, perhaps somewhat more exact, may be made with regards to the stresses due to axial and bending internal forces. Therefore, one-dimensional models are those that have such stresses in one direction, in a preferential manner to the others, which are orthogonal to it. Two-dimensional models are those that have such stresses in two orthogonal directions, in a preferential manner with respect to the third, which is orthogonal to both. Three-dimensional models are those where the stresses due to axial and bending internal forces are not predominant in any of the three orthogonal directions.

Numerical methods permit the analysis of the structures where, due to geometric complexity, the type of load or the behaviour of the materials, there are no analytical solutions or they are too complicated to be obtained.

Structural discretization and the types of elements employed should be suitable for the correct reproduction of the structural behaviour.

18.2 Geometric data

18.2.1 Effective flange width in linear members

In the absence of a more precise determination method, for cross-sectional verifications in T-beams, the stresses due to axial and bending internal forces are assumed to be uniformly distributed within a certain reduced width of the flanges, which is known as the effective width.

The effective width depends on the type of beam (continuous or simply supported), the manner in which the loads are applied, the ratio between flange thickness and beam depth, the presence or lack of brackets, beam length between points of zero moment, rib width and, finally, the distance between ribs in the case of a ribbed slab.

In practice, the effective width can vary along the length of the beam. Similarly, the effective width can vary in accordance with the state of cracking or yielding of the materials, and therefore may differ between service and ultimate situations.

The mentioned points of zero moment may, in practice, be considered fixed for all analysis hypotheses. They can also be obtained from the laws of moments due to permanent loads.

COMMENTS

In an approximate manner, it may be assumed that at the compression head, the effective width is equal to the rib width, plus one fifth of the distance between the points of zero moment, without exceeding the true flange width. If this approximation is employed, then the effective width can be considered constant throughout the span, including the segments close to the intermediate supports in the case of continuous beams. For edge beams, the effective flange width is the rib width, plus one tenth of the distance between the points of zero moment.

The effective flange width at the tension head can be considered equal to the rib width, plus eight times the flange thickness, or four times in the case of edge beams, without exceeding the true width.

In the case of T-beams with brackets with width b_c and height h_c (see figure 18.2.1), and exclusively for the purpose of calculating the effective width, the true rib width b_0 is substituted by another fictitious one b_e equal to the lesser of the following two values:

$$b_e = b_0 + 2\, b_c$$
$$b_e = b_0 + 2\, h_c$$

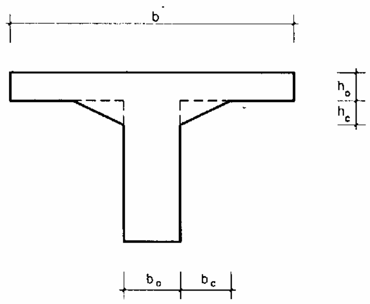


Figure 18.2.1

For box or T sections in bridges, the criteria for the definition of effective width as established by the Recommendations for Composite Road Bridges may be employed.

18.2.2 Design spans

Except justified cases, the design span of members shall be taken as the distance between support axes.

COMMENT

When the bearing dimension is large, the design span length can be obtained in a simplified way as the free span plus the element height.

18.2.3 Cross sections

18.2.3.1 General considerations

In most cases, the overall analysis of the structure may be performed by using the gross cross-sections of the elements. In certain cases, where greater precision is required in the verification of Serviceability Limit States, net or transformed cross-sections may be used in the analysis.

18.2.3.2 Gross cross section

The gross cross-section is that resulting from the true dimensions of the member, without deducting the spaces that correspond to the steel.

18.2.3.3 Net cross section

The net cross-section is that obtained from the gross cross-section by deducting the longitudinal spaces made in the concrete, such as ducts or grooves for prestressing steel or its anchorages.

18.2.3.4 Transformed cross-section

The transformed cross-section is that obtained from the net cross-section defined in 18.2.3.3 by taking into consideration the bonding effect of the adherent longitudinal reinforcement steel and the various types of concrete present.

COMMENTS

A reinforcement becomes bonded to the concrete when, due to adherence, any movement between the two materials is prevented. The bonding effect of a reinforcement with a concrete section is taken into account in the calculation by summing the area of the latter to that of the reinforcement multiplied by the so called "equivalence factor" $n = E_s/E_c$.

It should be remembered that if there are non-adherent active reinforcements, these cannot take part on the transformed section.

Concretes are transformed to a single type by multiplying the corresponding partial section width by the "equivalence factor" that results from dividing the concrete deformation module of the partial section by that of the reference concrete.

18.2.3.5 Cracked cross section

A cracked cross-section is that formed by the compressed area of concrete, plus the areas of the longitudinal steel, whether bonded prestressing or reinforcing steel, multiplied by the corresponding equivalence factor

COMMENTS

The position of the neutral axis in the cracked section in service situations should be calculated under the supposed elastic behaviour or the reinforcement and that of the compressed concrete (Appendix 9).

Article 19 Design methods

19.1 Basic principles

The conditions that, in principle, should be satisfied by every structural analysis are those of equilibrium and compatibility, taking into consideration the stress-strain behaviour of the materials.

In general, the compatibility conditions or stress-strain relations of materials are difficult to satisfy completely; therefore solutions may be adopted in which these conditions are only partially met, provided that they are in equilibrium and the appropriate ductility conditions are satisfied *a posteriori.*

19.2 Types of analysis

The overall analysis of a structure can be carried out in accordance with the following methods:

- Linear analysis
- Non-linear analysis
- Linear analysis with limited redistribution
- Plastic analysis

19.2.1 Linear analysis

This is based on the linear-elastic behaviour hypothesis of the component materials and the consideration of equilibrium in the structure without deformation. In this case, the gross concrete cross-section can be employed for the calculation of internal forces.

COMMENTS

This method is the most widely employed in the analysis of concrete structures. This approximation implies that the structural response is linear and that it accepts the reversibility of the deformations and the superposition of the effects caused by various actions. Due to the fact that, in general, the gross sections are used, this method requires knowledge of the geometry, but not necessarily that of the structure reinforcement steel.

19.2.2 Non-linear analysis

This type of analysis takes into account the mechanical non-linearity, that is, the non-linear stress-strain behaviour of the materials, and the geometric non-linearity or, in other words, the equilibrium in a structure in its deformed situation.

Non-linear behaviour makes the structural response depend on its load history. Therefore, in order to obtain the ultimate load, it is often necessary to proceed incrementally, covering the elastic, cracked and pre-failure ranges.

For a specific load level, non-linear analysis requires an iterative process which, after successive linear analyses, converges on a solution that will satisfy the equilibrium, stress and strain laws of materials and compatibility conditions. These conditions are checked out in a determined number of cross-sections, depending on discretization, which should be sufficient to guarantee adequate representation of the structural response.

Non-linear behaviour intrinsically implies that the superposition principle is not valid, and therefore, the safety format of Chapter III is not directly applicable in non-linear analysis.

COMMENTS

A structure exhibits non-linear behaviour when there is no proportionality between the action and the response. This type of behaviour is found in concrete structures in advanced phases of loading and, in some cases, in service conditions.

Concrete non-linearity is revealed by non-linear behaviour both in compression and tension, and by cracking when the tensile strength is exceeded.

Non-linearity in steel is revealed when this yields.

Imperfect bonding, the tension-stiffening effect (contribution of concrete under tension between cracks), the dowel effect of reinforcement in shear deformation, together with other secondary effects, also a cause of non-linearity in the behaviour of structural concrete.

In general, non-linear analysis requires a complete prior definition of the structure, both of its geometry and ~~the~~ reinforcement ~~steel~~ in all sections.

Generally speaking, non-linear analysis involves a different safety treatment to that described in Chapter III : It therefore frequently occurs that one works with the mean values of the material properties and progressively increases all or some of the loads until structural collapse takes place, thus obtaining an ultimate load value for each of the combinations under study.

19.2.3 Linear analysis with limited redistribution

The internal forces in this type of analysis are determined from those obtained by means of a linear analysis, just as described in 19.2.1, after which redistribution is performed satisfying equilibrium conditions.

Limited redistribution after a linear analysis requires suitable ductility conditions that guarantee the required redistribution by the adopted laws of internal forces.

COMMENTS

For those sections subject to bending with large amounts of tension reinforcement, which normally also requires reinforcement in the compression zone, or for sections that are subject to bending and heavy axial compression, the ductility conditions are usually reduced, unless special measures are taken, such as the use of transverse reinforcement in order to confine the concrete.

19.2.4 Plastic analysis

This analysis, based on plastic, elasto-plastic or rigid-plastic behaviour of the materials, satisfies at least one of the basic plasticity theorems: the lower bound, upper bound or uniqueness theorems.

COMMENTS

Plastic analysis is only permitted when sufficient ductility exists to guarantee the development of the assumed configurations.

This method is not allowed when it is necessary to take into consideration the second order effects.

Article 20 Prestressing structural analysis

20.1 General considerations

20.1.1 Definition of prestressing

Prestressing consists on applicating a controlled stress to concrete through the tensioning of steel tendons. The tendons must be made of high-tensile steel and may consist of wires, strands or bars.

This Instruction does not consider others forms of prestressing.

20.1.2 Types of prestressing

In accordance with the position of the tendons with respect to the cross-section, the prestressing may be:

a) Internal. In this case the tendon is situated within the concrete cross-section.

b) External. In this case the tendon is located outside the concrete’s cross-section and within its depth.

According the time with respect to the element casting, the prestressing may be:

a) With pre-tensioned reinforcement steel. The concrete is cast after the reinforcement steel has been tensioned and provisionally anchored to fixed elements. When the concrete has acquired sufficient strength, the reinforcement steel is released from its provisional anchorage and the force previously introduced into the reinforcement is then transferred to the concrete by bond.

b) With post-tensioned reinforcement steel. The concrete is cast before tensioning the prestressing steel, which is normally housed in ducts or sheaths. When the concrete has acquired sufficient strength, the steel is then tensioned and anchored.

From the point of view of tendon bonding, the prestressing may be:

a) Bonded. This is the case of prestressing with pre-tensioned or post-tensioned reinforcement where a material that provides adequate bonding between steel and concrete is injected after tensioning (Article 36.2).

b) Non-bonded. This is the case of prestressing with post-tensioned reinforcement where the steel is protected through the injection of materials that do not create ~~a~~ bond between steel and concrete (Article 36.3).

COMMENTS

In general, the term *prestressed concrete* is used to define all the described types, for a more precise definition of their characteristics it would be necessary to provide additional information in order to complete this description.

Tension oscillations due to variable loads, in normal situations of use, on the tendons of the various types of prestressing included in this Instruction, are sufficiently small to allow large initial tension. In the case of staying cables, tension variations due to variable loads are very significant and require specific limitations that are different to the ones given in this Instruction.

20.2 Prestressing force

20.2.1 Force limitation

The prestressing force P_0 should produce a stress of σ_{p0} to the tendons, which is no greater than the lesser of the following two values at any point:

$$0{,}75\, f_{p\,\max k}$$
$$0{,}90\, f_{pk}$$

where:

f_{pmaxk} Characteristic maximum tensile stress of prestressing steel.
f_{pk} Characteristic yield stress of prestressing steel.

This stress may be temporarily increased to the lesser of the following two values:

$$0{,}85\, f_{p\,\max k}$$
$$0{,}95\, f_{pk}$$

provided that when the steel is anchored into the concrete there is a suitable reduction of stress so that the previous paragraph's limitation is met.

COMMENTS

The objective of the stated limitations is to essentially reduce various types of construction risks, which depend on the precautions taken during execution and on the control that is carried out.

Independently of these risks, the uncertainty of the relaxation value of the prestressing steel increases with the prestressing load value.

20.2.2 Prestressing I in post-tensioned members

20.2.2.1 Evaluation of instantaneous losses of force

Instantaneous force losses are those that might occur during the prestressing operation and at the moment the tendons are anchored. They depend on the properties of the specific structural element. Their value at each cross-section is given by:

$$\Delta P_i = \Delta P_1 + \Delta P_2 + \Delta P_3$$

where:

- ΔP_1 Force losses at the specific section through friction along the prestressing duct.
- ΔP_2 Force losses at the specific section due to the draw-in of wedges at the anchorage points.
- ΔP_3 Force losses at the specific section due to elastic shortening of the concrete.

COMMENTS

In addition to the instantaneous losses due to friction, wedge draw-in or instantaneous deformation of the concrete, in special cases, it should be taken into consideration the losses due to other causes such as:

- mould deformation, in the case of prefabricated pieces;
- temperature differences between the tensioned reinforcement and the prestressed structure as a consequence of the concrete's treatment.
- instantaneous deformations at joints between precast elements in segmental construction.

The values of these losses should be determined experimentally.

20.2.2.1.1 Prestress losses due to friction

Theoretical prestress losses due to friction between the tendons and their sheaths or ducts depend on: the total angular variation α of the tendon's path between the specific section and the operating anchoring device that determines the stress at that section; the distance x between these two sections; the coefficient of friction μ for curves and K for straight lines. These losses are referred to the prestressing force P_0.

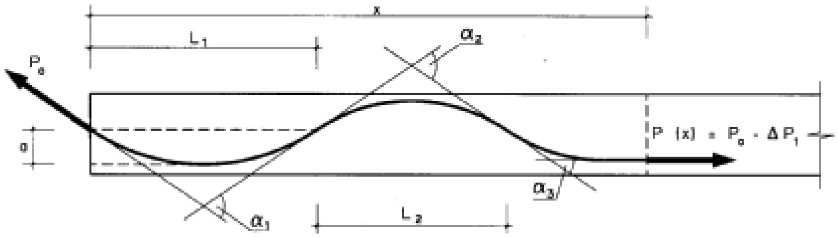
The friction losses at each section can be evaluated with the expression:

$$\Delta P_1 = P_0 \left[1 - e^{-(\mu\alpha + Kx)} \right]$$

where:

μ Coefficient of friction for curves between the tendons and their sheathing.

α	The sum of the absolute angular variation values (successive deviations), measured in radians, which describes the tendon at distance *x*. It should also be remembered that the tendon path could be a warped curve, then α should be evaluated in space.
K	Coefficient that denotes an unintentional angular displacement (per unit length) depending on the design layout (shape) of the tendon).
x	The distance in metres between the specific section and the operating anchoring device that determines the stress at that section (see figure 20.2.2.1).



If $a \leq 0.045\ L_1$ then α_1 may be taken as $\alpha_1 = \dfrac{8a}{L_1}$; being the error less than 5‰

$$\alpha = \sum_{i=1}^{x} \alpha_i = \alpha_1 + \alpha_2 + \alpha_3$$

α = Total angular displacement

α_i = Angular displacement in segment L_i

Figure 20.2.2.1.1

The data corresponding to the values of μ and *K* should be determined experimentally, by taking into account the prestressing procedure employed. If there is no real data, the experimental values sanctioned by practice may be employed.

COMMENTS

When, in the expression ΔP_1 the absolute value of the exponent *e* is less than 0.30, the stated formula may be employed in a linear fashion:

$$\Delta P_1 \approx P_0(\mu\alpha + Kx)$$

For internal prestressing, the values of μ basically depend on the state and nature of the surfaces in contact: sheaths or ducts in concrete, prestressing steel, any eventual lubrication, etc. When faces by a lack of experimental data, when all the elements (wires, strands, etc.) of the tendon are tensioned simultaneously, the values of μ given in Table 20.2.2.1.1.a. may be employed.

Table 20.2.2.1.1.a
Values of the coefficient of friction μ for curves

Arrangement of the prestressing steel in the sheaths	Surface condition of the prestressing steel	Nature of prestressing steels		
		Wires or drawn strands	Smooth rolled bars	Ribbed Rolled bars
1) Tendon consisting of several elements grouped together in the same steel sheath without any surface treatment	Without lubrication	0.21	0.25	0.31
	With light lubrication (soluble oil)	0.18	0.23	0.27
2) Tendon consisting of a single insulated element, in a sheath without treatment	Without lubrication	0.18	0.22	0.28
	With light lubrication (soluble oil)	0.15	0.20	0.24

NOTE: The values given in this table may increase by up to 0.10 if the tendon shows evidence of any rust on its surface, even when this is lubricated.

If the tendon's elements are tensioned separately, the values of μ are greater than those given in table 20.2.2.1.1.a, and they should be determined experimentally.

Regarding the coefficient K, it depends on the duct stiffness and other factors.~~,~~ It is the diameter that has greatest influence of the duct stiffness, so that, in an initial approximation, the values given in Table 20.2.2.1.1.b may be employed to determine the value of K from that of μ.

Table 20.2.2.1.1.b

Inside diameter of the duct [mm]	30	40	50	60	> 60
K/μ	0.016	0.012	0.009	0.007	0.006

It should be remembered that in the case of a partial release of prestressing, the coefficients of friction will be different and, in general, greater than those that appear in the process of increasing prestress. This fact may be taken into consideration in the design, deducing the new values for μ and K from experimental results.

For internal prestressing with non-bonded reinforcement steel, in accordance with the available experimentation and practical experience, the values given in Table 20.2.2.1.1.c may be taken for ~~those of~~ μ and K.

Table 20.2.2.1.1.c

	μ	K/μ
Individual strands with plastic protection	0.05 – 0.07	0.006 – 0.01

For those tendons used in external prestressing, the friction losses are concentrated in the saddles and are therefore strongly influenced by their characteristics. When specific data is lacking, the values given in Table 20.2.2.1.1.d may be employed, and which correspond to the case of multistrand tendons.

Table 20.2.2.1.1.d
Friction coefficient values for external prestressing

Features of saddles and tendon strands	μ	K/μ
1) Dry strands on steel tube	0.25 – 0.30	0.00
2) Greased strands on steel tube	0.20 – 0.25	
3) Dry strands on plastic tube	0.12 – 0.15	
4) Strands lined up on a plastic saddle	0.05 – 0.07	

20.2.2.1.2 Prestress losses due to wedge draw-in

In short, straight, post-tensioned tendons, the prestress loss due to wedge draw-in, ΔP_2, can be derived from the expression:

$$\Delta P_2 = \frac{a}{L} E_p A_p$$

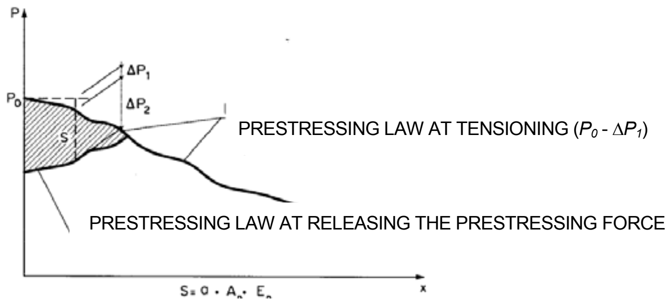
where:

a Wedge draw-in.
L Total length of the straight tendon.
E_p Modulus of elasticity of prestressing steel.
A_p Area of prestressing steel.

In the other cases of straight tendons and in all cases of curved ones, the evaluation of prestress loss through wedge draw-in should be evaluated by taking the in-duct friction into account. To do this, the possible variations in μ and K at releasing the prestressing force of the tendon can be considered, and compared to the values that appear on tensioning it.

COMMENTS

Losses due to wedge draw-in, in the case of post-tensioned steel, may be obtained from the prestressing force diagram along the length of the tendon, by reducing the force at the end corresponding to the operative anchorage device where the draw-in is being studied, up to a value such that the shortening of the tendon is equal to the wedge draw-in taking friction into account (see Figure 20.2.2.1.2).



S = AREA OF THE SHADED ZONE
a = DRAW-IN OF THE WEDGE

Figure 20.2.2.1.2

20.2.2.1.3 Prestress losses due to elastic shortening of the concrete

In the case of prestressing consisting of several tendons that are successively tensioned, as each tendon is stressed a further shortening of the concrete occurs, which in turn~~,~~ reduces the prestress on the tendons already anchored by an amount that is proportional to this shortening.

When the compression stresses at the level of the centroid of the presstressing steel being stressed are appreciable, the value of these losses, ΔP_3, can be calculated, if the tendons are stressed successively in a single operation, assuming that all the tendons undergo a uniform shortening, which is a function of the number n of the tendons that are successively stressed, through the following expression:

$$\Delta P_3 = \sigma_{cp} \frac{n-1}{2n} \frac{A_p E_p}{E_{cj}}$$

where:

A_p Total area of prestressing steel.
σ_{cp} Compression stress, at the level of the centroid of the prestressing steel, as produced by the force $P_0 - \Delta P_1 - \Delta P_2$ and the stresses caused by the internal forces acting at the moment of tensioning.
E_p Modulus of elasticity of prestressing steel.
E_{cj} Modulus of elasticity of concrete at age j corresponding to the tensioning.

COMMENTS

When the tendons are not successively tensioned in a single operation, but tensioning a group of them, and then another at a later date, the designer should estimate a suitable value of E_{cj} to be introduced into the expression of ΔP_3.

In any case, the load differences at the various tendons due to successive tensioning in the case of post-tensioning, and hence the loss ΔP_3., may be corrected on site by means of re-tensioning, or initial tensioning to decreasing loads of those tendons that are to be successively tensioned.

A more exact evaluation of the tension σ_{cp} may be performed by taking into consideration the modulus E_{cj} corresponding to the moment of introduction of each of the actions.

20.2.2.2 Time-dependent prestress losses

Time-dependent prestress losses are those that occur with time after anchoring the prestressing steel. These losses are essentially due to the shortening of the concrete due to shrinkage and creep and the relaxation of the prestressing steel.

Concrete creep and steel relaxation are influenced by the actual prestress losses themselves, and it is therefore essential to take this interaction into account.

Whenever a more detailed study of the interaction of these phenomena is not carried out, the time-dependent losses can be roughly estimated in accordance with the following expression:

$$\Delta P_{dif} = \frac{n\varphi(t,t_0)\sigma_{cp} + E_p \varepsilon_{cs}(t,t_0) + 0,80\Delta\sigma_{pr}}{1 + n\frac{A_p}{A_c}\left(1 + \frac{A_c y_p^2}{I_c}\right)\left(1 + \chi\varphi(t,t_0)\right)} A_p$$

where:

y_p Distance of the centroid of prestressing steel to that of the cross-section.
n Equivalence factor = E_p/E_c.
$\varphi(t,t_0)$ The creep coefficient for a loading age that is equal to the age of the concrete at the time of prestressing (t_0) (see 39.8).
ε_{cs} Shrinkage deformation that develops after the prestressing operation (see 39.7).
σ_{cp} Stress in the concrete fibre corresponding to the centroid of prestressing steel due to the prestressing action, self-weight and dead load.
$\Delta\sigma_{pr}$ Prestress loss due to relaxation at constant length. It can be evaluated using the following expression:

$$\Delta\sigma_{pr} = \rho_f \frac{P_{ki}}{A_p}$$

where ρ_f is the relaxation value at constant length and $t = \infty$ (see 38.9) and A_p is the total area of the prestressing steel. P_{ki} is the characteristic value of the initial prestressing force after instantaneous losses.

A_c The concrete cross-section area.
I_c The concrete cross-section moment of inertia.
χ Ageing coefficient. In a simplified way, for $t = \infty$, χ may be taken as being equal to 0.80.

COMMENTS

The precise value of prestress losses due to concrete creep and shrinkage and steel relaxation is difficult to obtain since the actual losses themselves reduce the compression stresses in the concrete and the tensile stress in the steel, resulting in a significantly complex interactive phenomenon.

In the article's simplified formula, the numerator represents the deformation due to shrinkage and creep in the concrete and relaxation in the steel, taking into account the previously mentioned interaction in a simplified fashion. The denominator represents the restraint of the time-dependent deformations due to the bonding prestressing steel. If the denominator is assumed to be equal to 1, this effect is ignored and the losses are over-estimated.

For a detailed evaluation, which is required for the evaluation of tensions and deformations in complex construction processes, the criteria given in Article 25 can be employed.

20.2.3 Prestress losses in pre-tensioned members

For pre-tensioned members, the losses to be taken into account between the time of tensioning and the transfer of prestress to the concrete are the following:

a) Wedge draw-in.
b) Relaxation at ambient temperature until the transfer.
c) Any additional relaxation of the steel due to the heating process, where applicable.
d) Any thermal expansion of the steel due to the heating process, where applicable.
e) Shrinkage prior to the transfer.
f) Instantaneous elastic shortening at the transfer.

Time-dependent losses after the transfer are obtained in the same manner as for post-tensioned elements, by employing the shrinkage and relaxation values that occur after the transfer.

COMMENTS

Wedge draw-in in pre-tensioned members produces a constant loss throughout the length of the prestressing steel due to the lack of friction, the value of which is obtained in the same manner as in the case for straight post-tensioned tendons that are short in length (20.2.2.1.2).

Loss by additional relaxation in the steel due, where applicable, to the heating process can be calculated in accordance with the information provided by the manufacturer of the steel. In the absence of this information, the relaxation value at 10^6 hours at 20ºC may be adopted as the sum of losses b) and c) of the article.

Loss d) may be evaluated by means of the expression:

$$K \ \alpha \ E_p \ (T_c - T_a)$$

where:

K	Experimental factor, to be determined at the factory, and in the absence of tests may be taken as K = 0.5.
α	Coefficient of thermal expansion of prestressing steel.
E_p	Modulus of elasticity of prestressing steel.
T_c	The maximum curing temperature during manufacture.
T_a	The average ambient temperature during manufacture.

The loss due to elastic shortening of the concrete when the pre-tensioned steel is released from its anchorages may be evaluated by taking into account the instantaneous deformation that is produced in the concrete at the centroid of prestressing steel by means of the following formula:

$$\sigma_{cp} \ \frac{A_p E_p}{E_{cj}}$$

20.3 The structural effects of prestressing

The structural effects of prestressing can be represented by using either a set of self-balancing equivalent forces or a set of imposed deformations. Both methods provide the same results

20.3.1 Modelling of prestressing effects using equivalent forces

The system of equivalent forces is obtained from the cable equilibrium and consists of:

- Forces and moments concentrated at the anchorages.
- Forces perpendicular to the tendons, resulting from curvature and changes in direction.
- Tangential forces due to friction.

The value of the forces and moments concentrated at the anchorage is deduced from the value of the prestressing force at these points, which is calculated in accordance with section 20.2, the cable geometry, and the geometry of the anchorage zone (see figure 20.3.1).

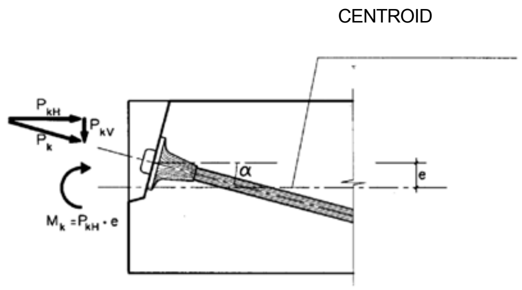


Figure 20.3.1

In the specific case of beams that are symmetrical with respect to a vertical plane, there will be horizontal and vertical components of the prestressing force, together with a bending moment at the anchorage, the expressions for which are as follows:

$$P_{k,H} = P_k \cos\alpha$$

$$P_{k,V} = P_k \, sen\alpha$$

$$M_k = P_{k,H} \, e$$

where:

α Angle formed by the tendon with respect to the axis of the element, at the anchorage.
P_k The force in the tendon, in accordance with 20.2.
e Eccentricity of the tendon with respect to the centroid of the section.

The transverse forces distributed along the length of the tendon, $n(x)$, are a function of the prestressing force and the curvature of the tendon at each point, $1/r(x)$. The tangential forces, $t(x)$, are proportional to the transverse forces through the friction coefficient μ, in accordance with:

$$n(x) = \frac{P_k(x)}{r(x)} \qquad ; \qquad t(x) = -\mu \, n(x)$$

COMMENTS

When representing the structure by means of bars connected by nodes, the forces distributed along the length of the tendon $n(x)$ and $t(x)$, may be taken as forces and moments that are either distributed along the length of the bars or concentrated at the nodes. In order to obtain the values it is necessary to establish the equilibrium of vertical and horizontal forces and moments, for the first case, in each section of tendon contained in a bar, and in the second, at each node.

20.3.2 Modelling of prestressing effects by means of imposed deformations

Alternatively, in the case of linear elements, the structural effects of prestressing can be introduced by the application of imposed deformations and curvatures, which, for each section are given by:

$$\varepsilon_p = \frac{P_k}{E_c A_c}$$

$$\left(\frac{1}{r}\right)_p = \frac{P_k e}{E_c I_c}$$

where:

ε_p The axial deformation due to prestressing.
E_c Modulus of elasticity of concrete.
A_c The concrete cross-section area.
I_c The concrete cross-section moment of inertia.
e Eccentricity of the prestressing force with respect to the concrete section's centroid.

20.3.3 Prestressing isostatic and hyperstatic forces

The structural internal forces due to prestressing, are traditionally defined by making a distinction between:

- Isostatic internal forces
- Hyperstatic internal forces

Isostatic internal forces depend on the prestressing force and the eccentricity of the prestressing with respect to the section's centroid, and can be analysed at the section level. Hyperstatic internal forces generally depend on the path of the tendon, the stiffness and support conditions of the structure and should be analysed at structure level.

The sum of the isostatic and hyperstatic internal forces due to prestressing is equal to the total internal forces produced by the prestressing.

When the Ultimate Limit State of bending is verified, in sections with bonded reinforcement, in accordance with the criteria established in Article 42, the design internal forces should include the hyperstatic part of the prestressing structural effect, using its value in accordance with the criteria provided in section 13.2. The isostatic part of prestressing is considered when evaluating the section's strength capacity, by taking into account the corresponding pre-deformation in the bonded prestressing steel.

COMMENTS

If prestressing is modelled by equivalent forces to simulate the structural effect of prestressing, the structural calculation will provide the total prestressing internal forces as results. If, on the other hand, prestressing is replaced with imposed deformations, the resulting internal forces of the structural calculation will only include the hyperstatic component. In this situation, in order to obtain the total stresses in the various concrete fibres of a section, the stress corresponding to the isostatic effect should be summed with that due to the hyperstatic internal forces.

The hyperstatic effect of the prestressing depends directly on the structure’s stiffness conditions, and this in turn depends on the load pattern applied. In Ultimate L~~l~~imit State, structure’s stiffness conditions are generally highly degraded, so that the hyperstatic effect of the prestressing can be dampened in consequence.

Article 21 Frames

21.1 General

When calculating internal forces in frame structures any of the methods given in Article 19 may be used.

21.2 Linear analysis

Linear analysis is especially suitable for Serviceability Limit States, although it is also valid for Ultimate Limit States in continuous beams and non-sway frames and for obtaining first-order internal forces in sway-frames, where the second-order effects are negligible, in accordance with Article 43.

COMMENTS

The use of linear analysis for obtaining the internal forces in ultimate limit state implies acceptance of the fact that critical sections possess a certain ductility which allows this assumed internal forces distribution without producing local failure.

21.3 Non-linear analysis

21.3.1 General

Non-linear analysis can be used for verification at both ultimate limit and serviceability limit states.

21.3.2 Analysis levels and models

The non-linear analysis models that are applicable to beams and frames can be grouped into three levels from greater to lesser complexity: micro-models for local studies, multi-layer models for non-linear sectional analysis, and those models based on the plastic hinge concept.

The use of sectional models or those based on the plastic hinge concept is sufficient for the analysis of this type of structures.

COMMENTS

Section level models enable to detect non-linear phenomena along the length of the members and are especially suitable for providing information about the effects of cracking in service, in addition to Ultimate Limit State behaviour.

When analysing beams or members that are subject to an axial force of known constant or slightly variable value with the loading level, it is advantageous to work with the moment-cuvature relationship, which constitutes the integrated sectional response. In the case of beams or supports subject to a reduced axial force, this ratio can be simplified by means of a tri-linear diagram that represents the elastic, cracked and pre-collapse phases.

21.3.3 Material behavioural models

In the case of frames and beams, uniaxial constitutive models for the materials are acceptable, i.e. models where the influence of stresses that are transversal to the main axis of the member are not taken into account in either stiffness or longitudinal strength.

COMMENTS

For instantaneous loads in concrete, the stress-strain diagram as defined by the following equation may be employed (see figure 21.3.3):

$$\sigma_c = \frac{k\eta - \eta^2}{1 + (k-2)\eta} f_{cm} \text{ for } \varepsilon_c \leq \varepsilon_{c,lim}$$

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}}$$

$$k = \frac{E_{ci}}{\frac{f_{cm}}{\varepsilon_{c1}}}$$

where:

σ_c	Concrete stress for a given value of ε_c.
ε_c	Concrete deformation.
ε_{c1}	Deformation at maximum concrete stress. ε_{c1}= 0.0022
$\varepsilon_{c,lim}$	Maximum concrete deformation, in accordance with Table 21.3.3.
E_{ci}	Tangent modulus of elasticity of the concrete, in accordance with Table 21.3.3.
f_{cm}	Average concrete strength.

Table 21.3.3

f_{ck} [N/mm^2]	25	30	35	40	45	50
E_{ci} [kN/mm^2]	32.0	33.5	35.0	36.5	37.5	38.5
$\varepsilon_{c,lim}$ [10^{-3}]	4.0	3.7	3.5	3.3	3.2	3.0

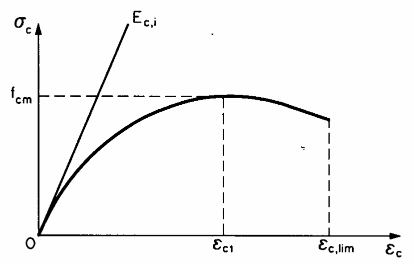


Figure 21.3.3

21.3.4 General method of non-linear analysis considering the second order effects

The general method of non-linear analysis considering the second-order effects takes into account simultaneously the non-linear behaviour of the materials, the equilibrium of the structure in its deformed configuration and the structural effects of the time-dependent deformations of the concrete.

In view of the interaction existing between the various causes of non-linearity, in order to realistically calculate the structural response and in particular the ultimate load, it is necessary to use stress-deformation diagrams that suitably represent the instantaneous and time-dependent behaviour of the materials in both service and under high load situations.

COMMENTS

The verification against buckling of frames, which is mandatory in certain cases in accordance with Article 43 of this Instruction, should be performed by means of a non-linear analysis considering the second order effects.It is recommended that a method based on the sectional analysis of a sufficient number of sections be employed for this verification. In addition, it should be checked that the load-bearing capacity of the various sections of the members is not exceeded.

21.3.5 Simplified methods of considering second-order effects

For sway frames requiring a theoretical second-order non-linear analysis, it might be sufficient to carry out a second-order elastic analysis, with a simplified representation of the reduction in stiffness due to the mechanical non-linearity.

COMMENTS

The methods of theoretical second-order elastic analysis mentioned in the article are iterative methods and incorporate the second order effects, through the updating of the structure's geometry, or by modifying the stiffness matrix using the stability functions or geometric matrix, or, in the case of buildings, by taking the p-delta effect, produced by the relative movement between floors, into consideration.

21.4 Linear analysis with limited redistribution

In the verification of ultimate limit states, a internal forces distribution that is in equilibrium with external forces may be obtained by applying redistribution (increases or reductions) to a linear design.

For a simplified approach to lintels of appreciably non-sway structures, except special justification, a redistribution of bending moments of up to 15% of the largest negative moment may be acceptable, provided that the depth of the neutral axis of the section over the support, when subjected to the redistributed moment in the ultimate limit state, is less than 0.45*d.*

COMMENTS

For a beam, the maximum permitted redistribution consists in substituting the diagram of moments obtained in a linear calculation by another that results from the displacement of the abscissa axis that passes through values of less than ±0.15M1 or ±0.15M2 (see figure 21.4.a). In the specific case where M1=M2=M, this redistribution is the equivalent of the vertical displacement of the theoretical curve by a value that does not exceed 0.15M (see figure 21.4.b).

The influence of the redistribution of moments should be taken into account in all aspects of the design: bending, shear, torsion, reinforcement steel anchorage and layout of reinforcement.

For a structure that is subject to various load hypotheses, the forces should be redistributed in each of them and then the envelope should be obtained.

In curved beams, this redistribution should be carefully analysed, since redistribution in bending could cause a sudden increase in the torsional moment, which could lead to a brittle type of failure before the redistribution of the bending moments has been totally completed.

In the case of rectangular sections, the depth of the neutral axis is related to the difference of the mechanical quantities of the tension and compression reinforcement steel and may be obtained in a simplified form with the following expressions:

- If $0.10 \leq \omega - \omega' \leq 0.18$

$$\frac{x}{d} = 1{,}1\,(\omega - \omega') + 0{,}06$$

- If $0.18 < \omega - \omega' \leq 0.42$

$$\frac{x}{d} = 1{,}45\,(\omega - \omega')$$

where $\omega = (A_s\ f_{yd})/(f_{cd}\ b\ d)$ and $\omega' = (A'_s\ f_{yd}\ /(f_{cd}\ b\ d)$ are, respectively, the mechanical quantities of tension reinforcement steel (A_s) and of compression (A'_s), *b* is the section width and *d* is the effective depth.

These same expressions are also applicable to T, TT or box sections, by taking *b* as the effective width of the compression head when the compression block in ultimate limit state is located in the compressed head, a condition which occurs when:

$$(\omega - \omega') \leq 0{,}85\ \frac{h_0}{d}$$

where h_0 is the compression head thickness.

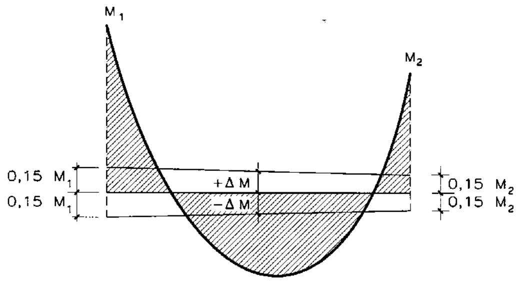


Figure 21.4.a

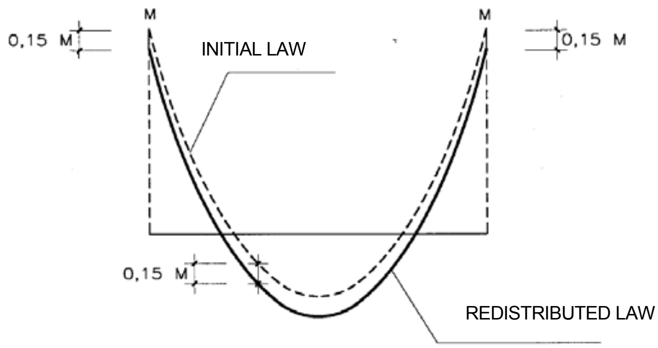


Figure 21.4.b

21.5 Plastic analysis

The application of this method is valid in ultimate limit state verifications and for those structures that are somewhat insensitive to second-order effects.

It should be verified that the plastic rotations required in the plastic hinges for the assumed mechanism are smaller than the limiting plastic rotations θ_{pi} of the affected structural elements.

Article 22 Slabs

22.1 General

This article is applicable to solid slabs, whether prestressed or not, subject to bending in both directions. Also included here are ribbed or hollow-core slabs provided that their behaviour, in terms of stiffness, can be related to that of a solid slab.

In order to treat a 2-way element as a slab, the minimum span must be greater than four times the average slab thickness.

This section includes slabs on continuous or point supports.

When calculating internal forces in slab structures any of the methods given in Article 19 may be used.

22.2 Linear analysis

Linear analysis is valid both for serviceability limit states and ultimate limit states.

Moments with pronounced gradients in localised areas (for example, under concentrated loads or supports) may be distributed over an area of suitable width, provided that the equilibrium conditions are fulfilled.

22.3 Non-linear analysis

Non-linear analysis may be employed for verifications of both serviceability limit and ultimate limit states.

Multi-layer models or moment-curvature relationships may be used, in combination with Kirchhoff's hypothesis.

Considerations in the behaviour of the concrete are the 2-way stress states, cracking in various directions, the orientation of the reinforcement steel and the contribution of the concrete under tension between cracks (tension-stiffening), the effect of which could be especially noticeable in serviceability limit states.

22.4 Simplified methods for slabs on point supports

22.4.1 General

The procedures described in this section may be applied to ultimate limit state internal forces design in structures that consist of solid or hollow-block reinforced concrete slabs reinforced with ribs in two perpendicular directions, which in general do not have beams to transmit the loads to the supports and directly rest on reinforced concrete columns either with or without capitals.

For vertical loads, the two simplified methods of obtaining internal forces as described in sections 22.4.3 and 22.4.4 may be used, in accordance with the type of slab.

For horizontal loads, only the method described in 22.4.4 is valid, provided that the stiffness characteristics of the slab elements are compatible with the phenomenon of moment transmission between support and slab, and that the structure complies with the specific conditions for the geometric arrangement of the supports in plan, with respect to their alignment and plan symmetry.

COMMENTS

The methods described in this section are the fruit of a long series of experimental results and the observation of suitable behaviour in various types of 2-way slabs.

22.4.2 Definitions

Capital: the widening of the upper end of a support that acts as the union between this and the slab. It may or may not be present.

Drop: area of a slab around a support or its capital which is widened, or in the case of a hollow-block slab is made solid with or without widening. It may or may not exist in solid slabs, but if it does, it can be used with a capital. It must be present in hollow-block slabs, and may be accompanied by a capital or not (figure 22.4.2.a).

Panel: a rectangular area of slab that is bounded by the lines joining the centres of four contiguous supports. For a given direction, a panel may be inner or outer (figure 22.4.2.b).

Inner panel: one that is located between two other panels in a given direction.

Outer panel: one that does not have a contiguous panel on one the sides in a given direction.

Corner panel: one that does not have contiguous panels on two of its sides.

Span: the distance between two parallel and consecutive lines of supports. Also used for naming each of the dimensions l_1 and l_2 of a panel.

Column strip: a strip of slab with a width equal to $0.25l_2$ on either side of a support. Column strips include beams, if any are present.

Middle strip: this is the strip between two column strips.

Equivalent frame: this is an idealised element adopted for designing a slab in a given direction. It consists of a line of supports and lintels with a cross-section equal to that of the area of slab bounded laterally by the axes furthest away from the panels adjacent to the specific line of supports, i.e. the zone includes a column strip and two half middle strips, one on each side.

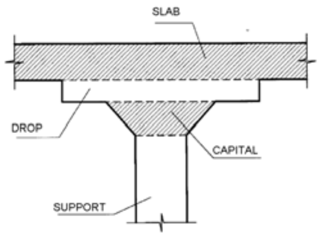


Figure 22.4.2.a

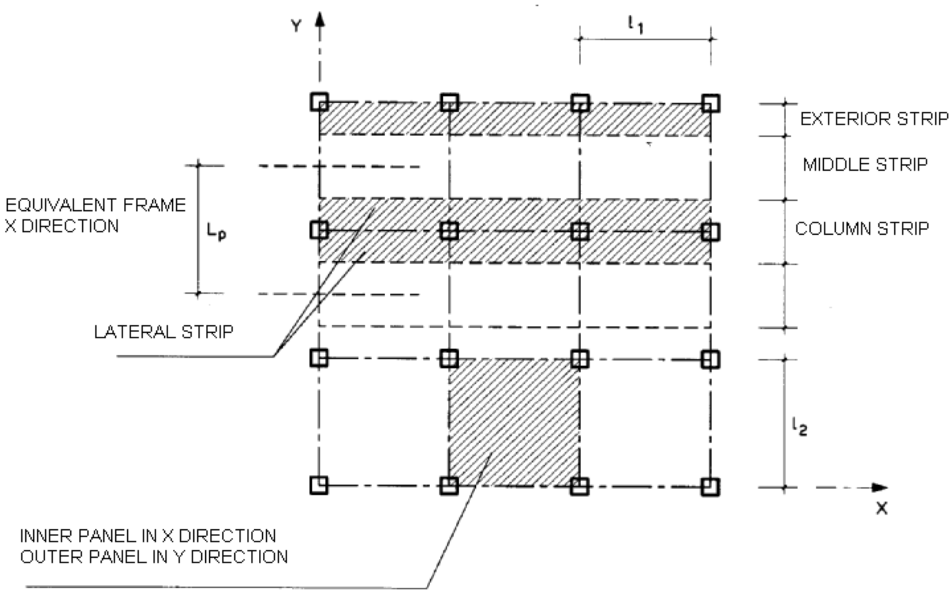


Figure 22.4.2.b

22.4.3 Direct method

For vertical loads, these slabs may be analysed by studying the equivalent frames, in each direction, which result whenever the limitations indicated in 22.4.3.1 are met.

The internal forces of the slab and supports in various equivalent frames may be determined by the simplified method of 22.4.3.2.

22.4.3.1 Scope

The following conditions must be met for this method to be applicable.

a) The grid defined in plan by the supports must be appreciably orthogonal.
An appreciably orthogonal grid is one where none of the supports deviate from

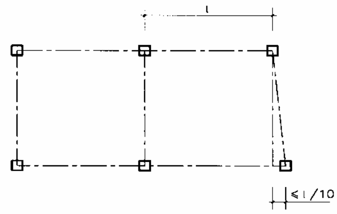


Figure 22.4.3.1

the line of axes defining the specific frame by more than 10% of the span perpendicular to it corresponding to the direction of deviation (figure 22.4.3.1).

b) The ratio of the longer side of the panel to the shorter shall not exceed 2.
c) The difference between the spans in consecutive bays shall not be greater than one third of that of the larger span.
d) The live load should be uniformly distributed and not greater than twice the permanent load.
e) There should be a minimum of three bays in each direction.

22.4.3.2 Internal forces in critical sections

The bending moments in the critical sections in each direction shall be determined from the moment M_0, which is defined as follows:

$$M_0 = \frac{(g_d + q_d)\, l_p\, l_1^2}{8}$$

where:

g_d Permanent design load applied on the specific panel.
q_d Imposed design load applied on the specific panel.
l_1 Distance between support axis in the direction for which the moments are calculated.
l_p Width of the analysed equivalent frame.

The moments of the critical sections in supports and spans are defined as a percentage of the moment M_0, in accordance with the values given in Table 22.4.3.2.

Table 22.4.3.2

	Case A	Case B	Case C
Negative moment in end support	30%	0%	65%
Positive moment in bay	52%	63%	35%
Negative moment in interior support	70%	75%	65%

Case A: Slab elastically restrained on edge supports.
Case B: Slab supported at the edge.
Case C: Slab fully restrained at both edges, or continuous over both supports (interior bay).

For interior supports, the adopted moment in the support shall be the greater of the two moments determined in accordance with both contiguous bays.

In the case of end bays of case A in Table 22.4.3.2, the edge beam should be designed to withstand the torsion due to a fraction of the moment considered at the edge of the slab.

In the case of end bays of case A in Table 22.4.3.2, the supports should be dimensioned to withstand the moment considered at the edge of the slab.

The interior supports shall be dimensioned to withstand an unbalanced moment defined in accordance with the following expression:

$$M_d = 0,07\,((g_d + 0,5\,q_d)\,l_p 1\, l_{11}^2 - g_d\, l_p 2\, l_{12}^2)$$

where:

l_{11}, l_{12} Dimensions l_1, corresponding to the bays adjoining the specific support.
l_{p1}, l_{p2} Dimensions l_p, corresponding to the bays adjoining the specific support.

Each support section, upper or lower, shall receive a fraction of the moment to withstand in proportion to its stiffness.

22.4.4 Equivalent frame method

For vertical and horizontal loads, these slabs may be analysed by studying the equivalent frames, in each direction, which result whenever the limitations indicated in 22.4.4.1 are met.

The specifications of the bars that represent the slab and supports may be obtained by means of the criteria given in 22.4.4.2.

The internal forces in the slab and supports can be determined by calculating the resulting equivalent frames for all load hypotheses, taking into consideration the most unfavourable combinations.

22.4.4.1 Scope

The fundamental hypothesis of this method lies in the non-interaction between equivalent frames. It should not therefore, be used in situations where such interactions are significant. Interaction between frames may be encountered in the following situations:

- Noticeable asymmetries (of geometry and stiffness) in plan or elevation.
- Presence of header beams.
- Notable sway structures.
- Existence of transverse stiffening elements (shear walls and stiff cores).
- Non-gravitational actions in non-uniform structures.
- Heavily unbalanced loads or spans.

22.4.4.2 Stiffness specifications of beams and supports in equivalent frames

The following criteria should be followed for vertical loads.

- When defining the moment of inertia of the beams that represent the slab, the gross moment of inertia corresponding to the total equivalent frame width should be employed, by taking into consideration the variation in stiffness that exists along the length of the bar.

- When defining the moment of inertia of the supports by taking into account the effect produced by the torsional restraint transversely conferred by the slab, an equivalent stiffness K_{eq} should be employed in accordance with the following expression:

$$\frac{1}{K_{eq}} = \frac{1}{K_c} + \frac{1}{K_t}$$

where:

K_c The gross support stiffness

K_t The stiffness of the torsional restraint elements (figures 22.4.4.2.a and b). A torsional support restraint element may be defined as that portion of slab with a width that is equal to the dimension c_1 of the support or capital, and a length that is equal to the width of the equivalent frame.

$$K_t = \Sigma \left(\frac{9\,E_c\,C}{l_2 \left(1 - \frac{c_2}{l_2} \right)^3} \right)$$

where:

E_c Modulus of elasticity of concrete.

C Torsional stiffness of the torsional restraint element.

l_2 Transverse dimension of the panel adjacent to the specific support.

c_2 Dimension of the specific support perpendicular to the equivalent frame.

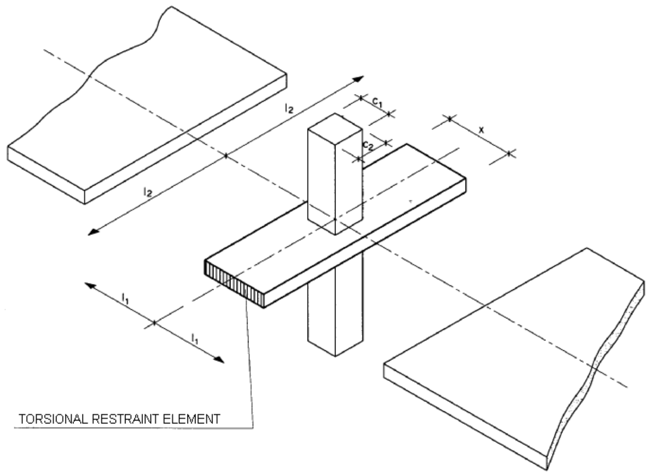


Figure 22.4.4.2.a

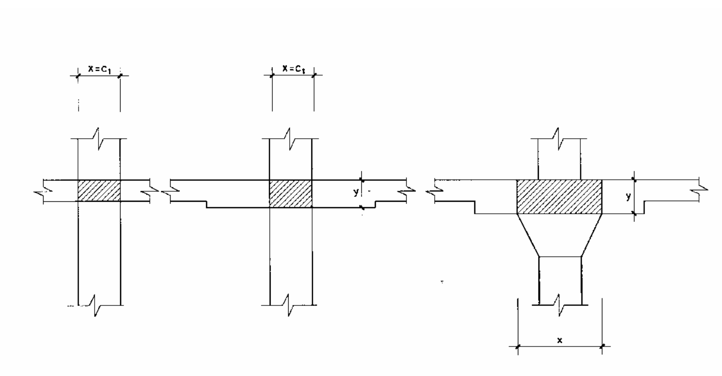


Figure 22.4.4.2.b

For interior frames, K_t is the sum of the torsional stiffness of the torsional restraint elements that exist on both sides of the specific support. For exterior frames, K_t is the torsional stiffness of the single torsional restraint element adjacent to the specific support.

The following expression can be employed as a definition of C (figure 22.4.4.2.b):

$$C = \left(1 - 0{,}63 \, \frac{x}{y} \right) x^3 \, \frac{y}{3} \quad , \; siendo \quad x < y$$

The following criteria should be followed for horizontal loads:

- When defining the moment of inertia of the beams that represent the slab, the gross moment of inertia corresponding to 35% of the equivalent frame width should be employed, by taking into consideration the variation in stiffness that exists along the length of the bar.

- The criteria given for vertical loads should be followed when defining the support moment of inertia.

22.4.5 Criteria for slab moment distribution

The moment distribution due to vertical loads on critical sections, supports and span, along the slab, as obtained by the procedures indicated in 22.4.3 and 22.4.4, should be performed in accordance with the criteria defined in Tables 22.4.5.a and b.

Table 22.4.5.a

Negative moments	On internal support	On external support
Column strip	75%	100%
Middle strip	25%	20%

Table 22.4.5.b

Positive moments	In both cases
Column strip	60%
Middle strip	40%

The moments due to horizontal loads should be absorbed throughout the width of the column strip.

22.4.6 Distribution criteria for moments between slab and supports

When a moment M_d acts on the union between slab and support, it is assumed that a fraction of this, equal to kM_d ,will be transmitted to the support by bending and the remaining fraction $(1 - k)\ M_d$ will be transmitted by tangential stresses. A simplified coefficient k can be taken as the values given in Table 22.4.6.

Table 22.4.6

c_1/c'_2	0.5	1.0	2.0	3.0
k	0.55	0.40	0.30	0.20

where:

c_1 is the dimension of the support in parallel to the eccentricity of the load or to the direction of the analysed equivalent frame.

c'_2 is the dimension of the support perpendicular to the eccentricity of the load or to the direction of the analysed equivalent frame in interior or corner supports and twice that dimension in outer supports.

In order ~~for it~~ to resist the partial moment kM_d , transmitted by bending, the slab should have the necessary reinforcement steel concentrated in a width that is equal to the column width plus 1.5 times the depth of the slab or drop on each side.

The fraction $(1 - k)\ M_d$ should be absorbed by torsion in the edge beam or torsional restraint element. Similarly, this fraction of the moment should be taken into account for the distribution of tangential stresses in the punching shear perimeter (Article 46).

COMMENTS

For dimensions c_1 and c_2, the values corresponding to the intersection between the support and the interior face of the slab should be adopted. Where there is a capital, dimensions c_1 and c_2 shall be those corresponding to the intersection of the capital with the lower slab face, with the faces of the capital forming an angle of no greater than 45º with the support axis.

In the case of supports with a circular or polygon cross sectional area, c_1 and c_2 shall be those corresponding to the square support of equal area.

Article 23 Membranes and shells

23.1 General

Shells are the surface structural elements that are thin with respect to their other dimensions and which are characterised by their resistant behaviour in three dimensions from a static point of view.

Shells are usually stressed by a combination of membrane and bending forces, and their structural response is basically influenced by their geometric shape, edge conditions and the nature of the applied load.

COMMENTS

In general, shells are supported by some or all of their edges on perimeter elements to which they transmit their loads, such as beams, arches or slabs. In other situations, the shells contain edge ribs, or internal ribs, with the usual objective of providing stiffness to the shell surface in order to prevent local deformations from reaching excessive values.

The edge conditions particularly influence the strength behaviour of the shells, behaviour that not only varies with the form of support, but also especially with the stress and deformation conditions of the edge elements.

Solutions that satisfy the equilibrium conditions can be found by only taking into consideration the membrane forces and ignoring the bending. However, this would involve heavy cracking, especially in the support zones with restricted movement. Therefore, at least on a local level, the bending should be taken into consideration during the analysis.

23.2 Types of structural analysis

During the determination of internal forces and deformation and for the study of shell stability, linear analysis can be employed, with all general elasticity hypotheses being applicable, together with any particular simplifications that have been shown by experience to hold for classic shell structure design. For such purposes, the concrete is assumed to have no reinforcement or cracking.

Shells that are subject to compression stresses should be analysed by taking into account any possible failure due to buckling. For this reason, elastic deformations and where applicable, those due to creep, temperature variation and shrinkage of the concrete, support settlement and any imperfections in the shell shape due to execution inaccuracies should be considered.

Non-linear analysis is also applicable. In this case, the effects of multi-axial stress states on the stiffness and strength of the concrete can also be included.

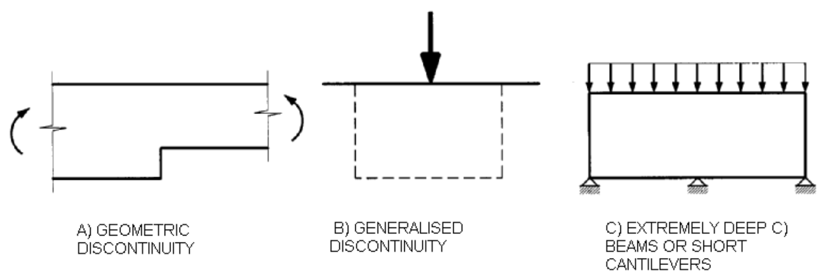
Plastic design should not be employed in internal forces determination, unless it can be reasonably shown that it is applicable to the particular case under consideration.

Article 24 D regions

24.1 General

D regions (discontinuity regions) are structures or parts of a structure where the general theory of bending is not valid, or in other words, where the Bernouilli-Navier or Kirchhoff hypotheses are not applicable. On the other hand, structures or parts of a structure where these hypotheses are met, are known as B regions.

D regions exist in a structure where abrupt changes of geometry occur (geometric discontinuities, figure 24.1.a), or in areas where concentrated loads and reactions are applied (static discontinuities, figure 24.1.b). Additionally, a D region may consist of an entire structure due to its shape or proportions (generalised discontinuity). Extremely deep beams or short cantilevers (figure 24.1.c) are examples of generalised discontinuities.



Figures 24.1.a, b and c

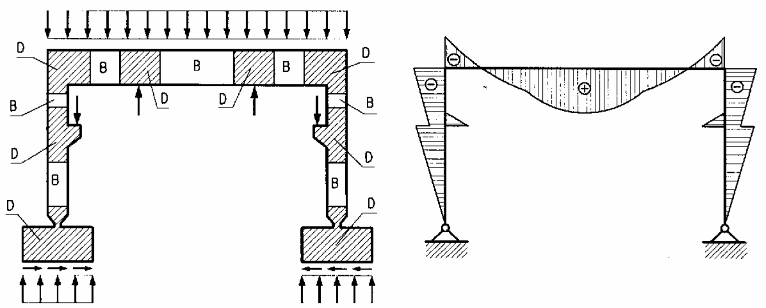


Figure 24.1.d

COMMENTS

Various zones that constitute D regions due to geometric or static discontinuities are identified in the structure shown in figure 24.1.d. On the other hand, the remaining zones of the structure are identified as B regions, where the general theory of bending and the Bernouilli-Navier hypotheses are valid.

24.2 Types of structural analysis

The following methods of analysis of internal forces are acceptable:

a) Linear analysis
b) Strut and tie method
c) Non-linear analysis

24.2.1 Linear analysis

The theory of elasticity can be adopted. The analysis provides the field of principal stresses and deformations. Stress concentrations, such as those that occur at corners or cavities, may be redistributed by taking into account the effects of cracking, reducing stiffness in the corresponding zones.

Linear analysis is valid both for behaviour in service and in ultimate limit states.

24.2.2 Strut and tie method

This method consists of replacing the structure or that part of it making up the D region with a structure of pinned members, generally in two dimensions but on some occasions three, which represents its behaviour. The compressed bars are defined as struts and represent the concrete compression. The tensioned bars are known as ties and represent the tensile forces within the reinforcement steel.

The model should be able to balance the external stresses that exist at the boundary of the D region, in the case of a zone of a structure, or the acting external loads and the support reactions, in the case of a structure with generalised discontinuity. This type of model, which assumes perfectly plastic behaviour, satisfies the requirements of the lower bound theorem of the theory of plasticity and, once it has been selected, the theorem of solution uniqueness.

This method allows verification of the structure's conditions at the ultimate limit state for the various combinations of actions as established in Article 13 if the conditions of the struts, ties and nodes are verified in accordance with the criteria set out in Article 40.

Serviceability limit state verifications, especially for cracking, are not explicitly carried out, but may be taken as being satisfied if the model is orientated with the results of linear analysis and the conditions for ties established in Article 40 are met.

COMMENTS

The struts and ties method is employed in D regions to explain the behaviour of linear elements that are subject to shear (flat lattice, figure 24.2.2.a) or to torsion (spatial lattice, figure 24.2.2.b).

It is recommended that the model be implemented by taking into consideration the stress distribution that is obtained by linear analysis, orientating the compressed struts in accordance with the principal compression stresses and the tensioned ties in accordance with the principal tensile stresses, adapting them to the possible reinforcement steel arrangements in the structural element.

If any experimental results are available that show suitable behaviour in serviceability and ultimate limit states, then as far as possible, the model should be orientated by taking into account the reinforcement steel arrangements in the tests.

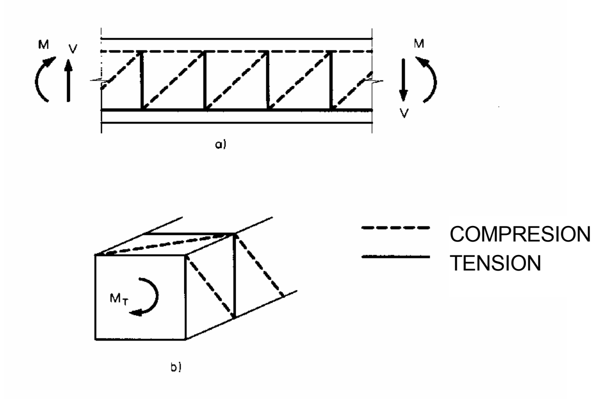


Figure 24.2.2.a and b

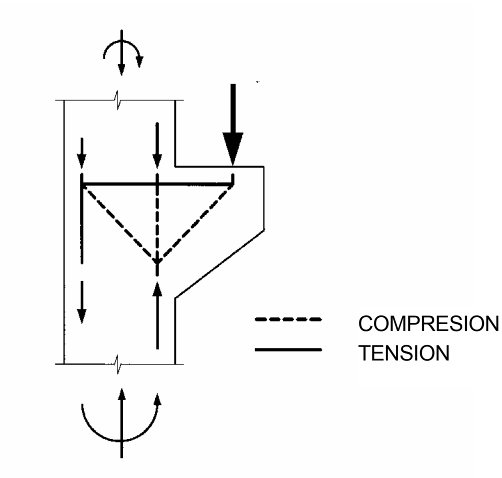


Figura 24.2.2.c

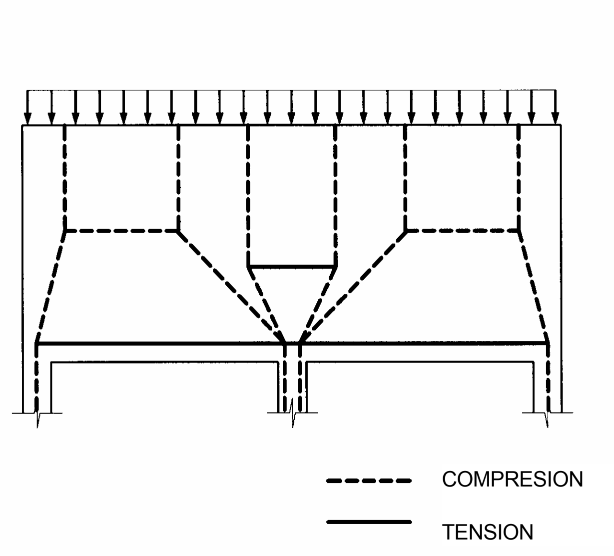
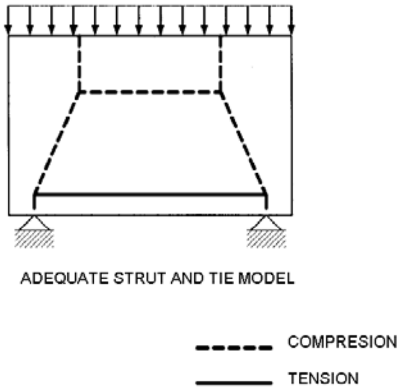
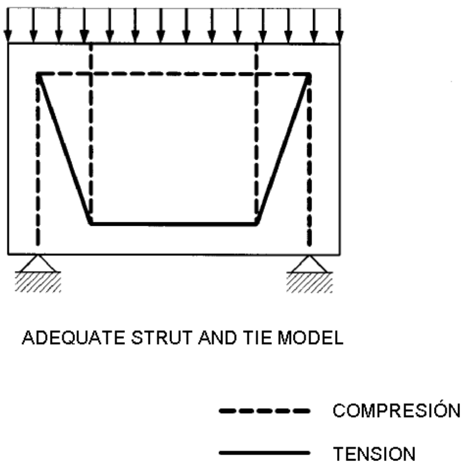
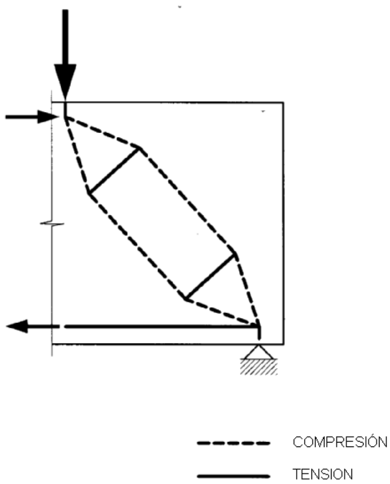


Figura 24.2.d







Figures 24.2.2.e, f and g

The use of statically determined models is preferred, in other words, those that do not require the implementation of compatibility conditions for obtaining the various forces of the bars. (figures 24.2.2.c and d).

Of all the possible bar models, those that require least deformation capacity in a plastic range are the most suitable, and hence, those where the tensioned ties have least length. (figures 24.2.2.e and f).

The model can be sufficiently refined so as to take into consideration the tensile stresses generated by the dispersion within the compression fields, which give rise to secondary reinforcement steel as shown in figure 24.2.2.g.

In general, the strut and tie method should be implemented by following the sequences given below:

- The definition of the external loads, the continuity forces and reactions of the D region.
- The study of the stress distribution by means of linear analysis. This calculation can be avoided when dealing with elements that have already been studied in the bibliography.
- The establishment of a bar model (struts and ties) where the external acting loads, the continuity forces and reactions of the existing supports are in equilibrium, following the various indications given in this section.
- The calculation of the bar forces employed in the model.
- Verification of the strut, tie and node conditions, in accordance with the criteria given in Article 40.
- When the previous verifications imply modifications to the dimensions of the bars, and hence, of the model's geometry, then the latter should be adjusted and a further calculation performed.

24.2.3 Non-linear analysis

For a more refined analysis, the non-linear stress-deformation relationships of materials under multi-axial load states can be taken into account by employing a suitable numerical method. In this case, the analysis is satisfactory for both serviceability and ultimate limit states.

Article 25 Time analysis

25.1 General considerations

Time analysis enables the structural effects of concrete creep, shrinkage and ageing and relaxation of the prestressing steel to be obtained. These effects may be time-dependent deformations and displacements, in addition to variations in the value or distribution of internal forces, reactions or stresses.

The analysis may be implemented on various levels:

I Step-by-step in time analysis or general method.
II Age-adjusted modulus method, which is also known as the ageing coefficient method.
III Simplified formulae that are based on the application of the ageing coefficient method to specific cases.

With respect to rheological phenomena, the general hypotheses that are valid for any of these procedures are as follows:

- Creep is considered to be independent of shrinkage.

- For each type of concrete in a section, average creep and shrinkage values may be adopted and therefore any differences that occur between different points can be ignored.
- Creep deformation is proportional to the stress that produces it (linear creep).
- Boltzmann's superposition principle is acceptable for the total assessment of deformation due to actions applied at various ages.
- These hypotheses are valid both for concrete under tension without cracking and under compression, provided that the compression stresses do not exceed 45% of the strength when the load is applied.

COMMENTS

Due to the rheological deformations of the concrete, variations are produced in the deformation state of the sections~~,~~ and, therefore, increments in the deflections and movements.

In addition, due to the different time-dependent behaviour of the concrete and of the steel and their joint work, through time, variations in the stress state of both materials and in the actual force distribution itself among the various sections are produced.

The main causes of the stress and force variations are as follows:

- The existence of materials with different rheological properties working together, both in the same section and through the structure as a whole.
- Modifications in the longitudinal structural scheme, in the cross section or in the support conditions due to evolving construction processes or later operations on the structure.

The accuracy of the evaluation procedures for the time-dependent effects should be consistent with the reliability of the data available for the description of these phenomena and with the significance in the limit state under consideration.

The type of analysis that should be carried out will depend on the type of structure (sensitivity against time-dependent behaviour) and the type of problem that is to be dealt with: the control of a constructive evolutionary process or the analysis of a structure at $t = \infty$.

For the analysis at $t = \infty$, most structures may be studied at Level III. Among these may be mentioned normal building structures, slab bridges constructed on centering or span by ~~to~~ span etc.

Level II analysis is sufficient for determining the long term distribution of forces and stresses for many structures since it provides very good results when compared with the general method. In particular, this analysis level may be applied to bridges with composite sections (precast beam decks with *in situ* upper slab, for example) since their sensitivity to time-dependent effect is quite important.

Level I analysis may therefore be reserved for the control of constructive evolutionary processes where the control of deflection and movement in the structure, or the stress state of ancillary elements (ties ,~~and~~ provisional props, etc.). Within this group, mention may be made of bridges constructed by the free cantilever method or by other constructive methods with a marked evolutionary character.

25.2 General Method

In applying the general, step-by-step method, the following hypotheses should be applied:

a) The constitutive equation for concrete in time is:

$$\varepsilon_c(t) = \frac{\sigma_0}{E_c(t)} + \varphi(t,t_0)\frac{\sigma_0}{E_c(28)} + \sum_{i=1}^{n}\left(\frac{1}{E_c(t_i)} + \frac{\varphi(t,t_i)}{E_c(28)}\right)\Delta\sigma(t_i) + \varepsilon_r(t,t_s)$$

In this equation, the first term represents the instantaneous deformations due to a stress applied at t_0. The second term represents the creep due to this stress. The third term represents the sum of the instantaneous and creep deformations due to the variation in stresses occurring at instant t_i. Finally, the fourth term represents the shrinkage deformation.

b) The various types of steel are considered to behave linearly under instantaneous loads. For prestressing steels with stresses greater than $0.5f_{pmax}$ relaxation shall be taken into account, together with the fact that this is produced at variable deformation.

c) It is considered that perfect bonding exists between the concrete and the bonded steel, and between the various concretes that might exist within the section.

d) In the case of linear elements, the hypothesis of section plane deformation is accepted as being valid.

e) Equilibrium conditions shall be verified at any section level.

f) Equilibrium shall be verified at the structure level, taking the support conditions into account.

COMMENTS

Concrete creep at each point depends on its stress history, which in general, is not known and also varies with time. This is why a step-by-step process is implemented, dividing time into intervals and carrying out a structural analysis on each one that satisfies the conditions of equilibrium, compatibility and the constitutive equations of the materials at the moment under consideration. The stress variation in the concrete produced along this interval is obtained as a result of this calculation. Once this is known, it is then possible to calculate the deformation that this will produce in the following time interval and in succession.

25.3 Coefficient of ageing method

When applying the coefficient of ageing method, the hypotheses given in 25.2 are valid, but with the following additional simplifications:

a) The deformation produced by stress variations with time in the concrete may be taken as being equal to that which such an increase in stress would produce if applied at an intermediate moment of time and maintained constant.

$$\int_{\tau=t_0}^{t} \left(1+\varphi(t,\tau)\right)d\sigma(\tau) = \left(1+\chi(t,t_0)\varphi(t,t_0)\right)\Delta\sigma_{t_0\to t}$$

where χ is the ageing coefficient. The value of χ may be determined at any given moment, by means of a step-by-step calculation and may be taken as being equal to 0.80 for $t = \infty$.

b) Relaxation at variable deformation may be evaluated in a simplified manner at infinite time as being the relaxation at constant length, multiplied by a reduction factor of 0.80.

COMMENTS

The ageing coefficient method enables the variations in stress, deformation, forces and movements due to the time-dependant behaviour of the concrete and the prestressing steel at infinite time to be calculated and avoiding time discretization. In particular, on a section level, the increments in axial and curvature deformation due to creep, shrinkage and relaxation may be determined by means of expressions that are relatively simple compared to the complexity of the phenomenon and which may be found in specialised literature.

25.4 Simplifications

From the expressions given in 25.3, it is possible to deduce a series of simplified formulas by ignoring the difference between transformed sections at t = 0 and $t = \infty$, or by ignoring the hyperstatic effect of the imposed deformations, as the case may be.

COMMENTS

An important application of this type is the formula for calculating the forces at infinite time for those structures that undergo changes in the support conditions (span-to-span construction, free cantilever construction, movements at supports, etc.). In these cases, it is possible to estimate the internal forces distribution at $t = \infty$ through the employment of the following simplified expression:

$$S_\infty = S_0 + (S_c - S_0)\frac{\varphi(\infty, t_0) - \varphi(t_c, t_0)}{1 + \chi\varphi(\infty, t_c)}$$

where:

S_0 The internal forces at the end of the constructive process.
S_c The internal forces that are obtained if the structure is constructed on centering.
t_0 Concrete age on applying the load.
t_c The age of the concrete for which the support conditions are changed.

Another example is the formula in 20.2.2.2 for the calculation of the prestressing time-dependent losses.