## SECTION 4 DESIGN OF SECTIONS AND STRUCTURAL ELEMENTS

## MATERIALS DATA FOR THE PROJECT

## CHAPTER VIII

## Article 38 Steel properties

## $38.1 \quad$ General

The steel employed for reinforcement shall meet the requirements prescribed in Article 31.

Prestressing reinforcement shall consist of steel that meets the requirements prescribed in Article 32.

### 38.2 Characteristic stress-strain diagram for reinforcing steel

The characteristic stress-strain diagram is that adopted as a basis for design, and is associated in this Instruction with $5 \%$ of the lowest stress-strain diagrams.

The characteristic stress-strain diagram of steel in tension is that with the property where the stress values corresponding to strains no greater than $1 \%$ have a confidence level of 95 per cent in relation to the corresponding values obtained in tensile tests performed in accordance with UNE 7474-1:92.

The same diagram may be used for compression and for tension.
When precise experimental data is lacking, the characteristic diagram may be assumed to have the shape of Figure 38.2, and this may be taken as a characteristic diagram if the grade values for yield stress given in Article 31 are adopted.


Figure 38.2 Characteristic stress-strain diagram for reinforcing steel.
In all cases, the compression branch is symmetrical with the tension branch with respect to the origin.

## COMMENTS

Knowledge of the characteristic diagram for the steel enables the dimensioning of the sections subjected to normal stresses (flexion and compression) with greater accuracy and economy than when only the yield stress value is known. It is therefore recommended that the steel producers establish and guarantee this diagram for each of the types that they supply, with the aim of classifying them as characteristic diagrams.

In order to establish the diagram and verify it through acceptance tests, the determination of the stresses that correspond to the following strains is accepted as sufficient. 1, 2, 3, 4, 5, 6, 8 and 10 per 1,000.

The diagram provided in the article is a suitable simplification of the real diagram for steel. If, due to the requirement for greater accuracy, curvilinear diagrams are employed (from experimentally obtained diagrams), the yield stress obtained from the tension test yield stress values in accordance with UNE 7474$1: 92$ may be adopted as the characteristic value.

When experimental data is lacking, $\varepsilon_{\text {máx }}$ may be taken as being 0.08 or 0.05 depending on whether it is B 400 S or B 500 S steel respectively, while $f_{\text {máx }}$ may be taken as being $1.05 f_{y k}$ (figure 38.2).

### 38.3 Design strength of passive reinforcing steel

The design strength of steel $f_{y d}$ shall be taken as the value:

$$
f_{y d}=\frac{f_{y k}}{\gamma_{s}}
$$

where $f_{y k}$ is the characteristic yield stress and $\gamma_{s}$ is the partial safety factor as defined in Article 15

Where a reduced level of control (inspection) for steel is used (90.2), the following value shall be taken as the design strength of the steel:

$$
f_{y d}=\frac{0,75 f_{y k}}{\gamma_{s}}
$$

The given expressions are valid for both tension and compression.
When steels of different yield stresses form part of the same section, each one shall be taken into consideration in the design with its own diagram.

## COMMENTS

It should be remembered that in sections that are subjected to compression, the failure strain of the concrete limits the exploitation of the design strength of steel to the stress value corresponding to this strain on the employed steel diagram.

### 38.4 Design stress-strain diagram for reinforcing steel

The design stress-strain diagram for reinforcing steel (under tension or compression) is obtained from the characteristic diagram by means of an oblique affinity with a ratio of $1 / \gamma_{s}$ parallel to Hooke's line.

When the diagram in figure 38.2 is used, the design diagram in Figure 38.4 is obtained; this shows that after $f_{y d}$ the second branch may either be considered to have a positive slope, obtained by oblique affinity from the characteristic diagram, or it may be considered horizontal, which in general terms is sufficiently precise.

Other design diagrams may be used, provided that the results they provide are sufficiently guaranteed by experience.


Figure 38.4 Design stress-strain diagram for reinforcing steel.

## COMMENTS

The strain of steel under tension is limited to a value of $1 \%$ and that of compression to $0.35 \%$, in accordance with that given in 42.1.3.

In general, it is sufficient to use the bilinear design diagram with the horizontal branch as from the yield stress and taking the modulus of elasticity of reinforcing steel, $E_{s}=200.000 \mathrm{~N} / \mathrm{mm}^{2}$.

### 38.5 Characteristic stress-strain diagram for prestressing reinforcement

As characteristic stress-strain diagram of prestressing reinforcement (wire, bar or strand), the one given by the producer may be adopted up to a strain of at least $\varepsilon_{\mathrm{p}}=0.010$, associated with a confidence level of 95\%.

If this guaranteed diagram is not available, then the one shown in Figure 38.5 may be used. This diagram consists of a first straight branch having a slope of $E_{p}$ and a second curved branch beyond $0.7 f_{p k}$, which is defined by the following expression.

$$
\varepsilon_{p}=\frac{\sigma_{p}}{E_{p}}+0,823\left(\frac{\sigma_{p}}{f_{p k}}-0,7\right)^{5} \quad \text { para } \quad \sigma_{p} \geq 0,7 f_{p k}
$$

where $E_{p}$ is the modulus of elasticity of prestressing steel as defined in 38.8.


Figure 38.5 Characteristic stress-strain diagram for prestressign reinforcement.

### 38.6 Design strength of prestressing reinforcement

Design strength of prestressing reinforcement is taken as being:

$$
f_{p d}=\frac{f_{p k}}{\gamma_{s}}
$$

where $f_{p k}$ is the characteristic tensile stress and $\gamma_{s}$ is the partial safety factor as defined in Article 15.

### 38.7 Design stress-strain diagram for prestressing reinforcement

The design stress-strain diagram for prestressing reinforcement is obtained from the corresponding characteristic diagram by means of an oblique affinity with a ratio of $1 / \gamma_{s}$ parallel to Hooke's line (see Figure 38.7.a).

As a simplification, $\sigma_{p}=f_{p d}$ may be taken beyond $f_{p d}$ (see figure 38.7.b).


Figure 38.7.a. Design stress-strain diagram for prestressing reinforcement.


Figure 38.7.b Design stress-strain diagram for prestressing reinforcement.

### 38.8 The modulus of elasticity of prestressing steel for active reinforcement

A value of $E_{p}=200.000 \mathrm{~N} / \mathrm{mm}^{2}$ shall be adopted for the modulus of elasticity of prestressing steel for wires or bars, except in the case where there is experimental justification.

In strands, the initial and reiterative values set by the producer may be adopted, or they can be determined experimentally. In the characteristic diagram (see 38.5), the value of the reiterative modulus should be used. If there are no previous experimental values, the value $E_{p}=$ 190,000 N/mm2 may be adopted.

Verification of elongation during tensioning requires the use of the initial modulus value, as determined by experimentation.

## COMMENTS

In strands, the initial modulus of elasticity, in other words, of the first load, is less than the reiterative modulus, after successive unloading and loading, with a difference of approximately $10 \mathrm{kN} / \mathrm{mm}^{2}$ or greater.

### 38.9 Relaxation of prestressing reinforcement

The relaxation of steel at constant length $\rho$, for an initial stress of $\sigma_{p i}=\alpha f_{\text {max }}$, where the fraction $\alpha$ lies between 0.5 and 0.8 and for time $t$, may be estimated using the following expression:

$$
\log \rho=\log \frac{\Delta \sigma_{p}}{\sigma_{p i}}=K_{l}+K_{2} \log t
$$

where:
$\Delta \sigma_{p} \quad$ The loss of stress through relaxation at constant length after time t , in hours.
$K_{1}, K_{2}$ are coefficients that depend on the type of steel and the initial stress (figure 38.9).


Figure 38.9
The producer shall provide the values of relaxation after 120 hours and 1000 hours for initial stresses of $0.6,0.7$ and 0.8 times $f_{\max }$ at a temperature of $20 \pm 1^{\circ} \mathrm{C}$, and shall guarantee the 1,000 hours value for $\alpha=0.7$.

Using these relaxation values, the coefficients $K_{1}$ and $K_{2}$ may be obtained for $\alpha=0.6$, 0.7 and 0.8.

The relaxation for other values of $\alpha$ may be obtained by linear interpolation, accepting $\alpha$ $=0.5, \alpha=0$.

The final value $\rho_{f}$ should be taken as being the value for the estimated design life of the structure in hours or the value for 1.000 .000 hours if this is not available.

## COMMENTS

When experimental data is not available for the evaluation of the losses due to relaxation, these may be estimated as shown below using the procedure explained in the article.

The relaxation after 1,000 hours ( $\rho_{1000}$ ) for initial stresses equal to $0.6,0.7$ and 0.8 of $f_{\text {máx }}$ may be obtained from Table 38.9.a. The values of the table indicate the percentage loss of the initial stress.

Table 38.9.a

| Type of reinforcement | $\mathbf{0 . 6 f}_{\text {máx }}$ | $\mathbf{0 . 7 f}_{\text {máx }}$ | $\mathbf{0 . 8 f}_{\text {máx }}$ |
| :--- | :---: | :---: | :---: |
| Wires and strands | 1.0 | 2.0 | 5.5 |
| Bars | 2.0 | 3.0 | 7.0 |

The variation in relaxation up to 1,000 hours may be estimated from the percentages given in Table 38.9.b.

Table 38.9.b

| Type of reinforcement | $\mathbf{0 . 6 f}_{\text {máx }}$ | $\mathbf{0 . 7 f}_{\text {máx }}$ | $\mathbf{0 . 8 f}_{\text {máx }}$ |
| :--- | :---: | :---: | :---: |
| Wires and strands | 2.9 | 5.8 | 16.0 |
| Bars | 5.8 | 8.7 | 20.4 |

In order to estimate the relaxation for times greater than 1,000 hours and up to infinite time, the following expression may be employed:

$$
\rho(t)=\rho_{1000}\left(\frac{t}{1000}\right)^{k}
$$

where:
$\rho(t) \quad$ relaxation after thours.
$\rho_{1000}$ relaxation after 1,000 hours.
$\rho_{100}$ relaxation after 100 hours.

$$
k=\log \left(\frac{\rho_{1000}}{\rho_{100}}\right)
$$

The final relaxation at constant length for various values of initial stress may be obtained from Table 38.9.c. The values of the table show the percentage loss of the initial stress.

| Table 38.9.c |
| :--- |
| Time in hours 1 5 20 100 200 500 <br> 1000       <br> evolution of relaxation losses up to 1,000 <br> hours, as percentaga. 25 45 55 70 80 90 100 |

The relaxation values are highly temperature sensitive. When carrying out processes where the temperature varies with respect to normal values (steam curing, etc), then experimental data at these other temperatures should be available.

### 38.10 Fatigue characteristics of reinforcement

The variation in maximum stress due to fatigue load should be less than the fatigue limit values that are given in Table 38.10.

Table 38.10 Fatigue limit for reinforcement

| Type of steel | Fatigue limit $\Delta \sigma_{D}\left[\mathrm{~N} / \mathrm{mm}^{2}\right]$ |  |
| :--- | :---: | :---: |
|  | Direct bonding | Bonding within the <br> steel sheaths |
| Reinforcing steel |  | - |
| $-\quad$Bars <br> wire fabrics (welded mesh <br> fabrics). | 150 | - |
| Prestressing reinforcement | 100 |  |
| $-\quad$ Wires | 150 | 100 |
| $-\quad$ 7-wire strands | 150 | 100 |
| - | - | 100 |

In the case of bent bars, when specific and representative experimental results are lacking, the fatigue limit indicated in Table 38.10 shall be reduced in accordance with the following criterion:

$$
\Delta_{\sigma_{D, \text { red }}}=\left(1-3 \frac{d}{D}\right) \Delta_{\sigma_{D}}
$$

where:
d is the bar diameter
$D \quad$ is the bend diameter
In the case of vertical stirrups having a diameter of less than, or equal to, 10 mm , no reduction in the fatigue limit is required.

## COMMENTS

Welded bar unions should be avoided as far as is possible in the case of fatigue stress. Where these type of unions are unavoidable, butt joints with complete penetration are acceptable, and in a situation where specific and representative tests are not available, the fatigue limit values given in Table 38.10, reduced by $50 \%$ shall be taken.

The acceptance of welds in the assembly of links is not recommended.

### 38.11 Fatigue characteristics of anchoring and coupling devices for prestressing reinforcement.

Anchoring and coupling devices shall, whenever possible, be located in sections where the variation in stresses is minimal.

In general, the fatigue limit for such elements is lower than that of the reinforcement and shall be supplied by the producer after specific and representative tests have been performed.

## Article 39 Concrete properties

### 39.1 Definitions

Project characteristic strength, $f_{c k}$, is the value adopted in the project for compressive strength as the basis for design purposes. It is also known as the specified characteristic strength or design strength.

The real characteristic strength, $f_{c}$ real, on site is the value corresponding to the 5 per cent fractile in the distribution of compression strength of the concrete placed on site.

The estimated characteristic strength, $f_{c}$ est, is the value that estimates or quantifies the actual characteristic strength on site based on a finite number of results from standardised compressive strength tests carried out on specimens taken on site. It may be abbreviated to characteristic strength.

The estimated characteristic strength shall be determined in accordance with 88.4.

If there are not any test results available, it may be assumed that the lower characteristic tensile strength $f_{c t, k}$ (corresponding to the 5 per cent fractile ( $5 \%$-quantile)) is a function of the project characteristic compressive strength $f_{c k}$, according to the expression:

$$
f_{c t, k}=0,21 \sqrt[3]{f_{c k}^{2}}
$$

The mean tensile strength $f_{c t, m}$ and upper characteristic tensile strength (corresponding to the 95 per cent fractile) $f_{c t, k 0.95}$ may be estimated, in the absence of test results, by using:

$$
\begin{aligned}
f_{c t, m} & =0,30 \sqrt[3]{f_{c k}^{2}} \\
f_{c t, k 0,95} & =0,39 \sqrt[3]{f_{c k}^{2}}
\end{aligned}
$$

In all these formulae, $f_{c t, k}, f_{c k}, f_{c t, m}$ and $f_{c t, k 0,95}$ are expressed in $\mathrm{N} / \mathrm{mm}^{2}$.
In this Instruction, the term characteristic tensile strength always refers to the lower characteristic tensile strength $f_{c t, k}$, unless otherwise indicated.

## COMMENTS

The definitions are established by taking the following into consideration:

- The strength of the concrete placed on site is a random variable with a distribution function that, in general, is unknown, but where the $5 \%$ fractile is the real characteristic strength.
- The project characteristic strength $f_{c k}$ is a lower specification limit, which establishes the condition that each batch placed on site shall have a strength equal to, or greater than, $f_{c k}$.
- The scope of the control, established in article 88 , is precisely to guarantee that a maximum of $5 \%$ of the batches that form the lot under control have a strength equal to, or less than, the project characteristic $f_{c k}$.
- In most cases, it may be assumed that the concrete strength behaves in accordance with a Gaussian distribution, in which the real characteristic strength $f_{\text {creal }}$ is given by the expression:

$$
f_{c, \text { real }}=f_{c m}(1-1,64 \delta)
$$

where $f_{c m}$ is the average strength and $\delta$ is the population variation coefficient..

The expressions in the article which give the average strength, the upper and lower concrete characteristics tensile strength in function of the characteristic compression strength are valid for ages equal to, or greater than 7 days, under normal curing conditions, and may give highly unreliable results for lower ages.

In the case where the splitting tensile strength or the flexural tensile strength has been determined experimentally, the comments given in 30.3 explain how to obtain the concrete tensile strength from these values.

### 39.2 Grading (designation) of concretes

Concretes shall be designated in accordance with the following format (which shall be used on the project drawings and in the Project Specification)

$$
T-R / C / T M / A
$$

where:
$T \quad$ A code which is HM for mass concrete, HA for reinforced concrete and HP for prestressed concrete.
$R \quad$ specified characteristic strength, in $\mathrm{N} / \mathrm{mm}^{2}$
C Letter indicating type of consistency, as defined in 30.6.
TM Maximum aggregate size in millimetres, as defined in 28.2.
A Environmental description (exposure class), in accordance with 8.2.1.
With regards to the specified characteristic strength, the following series is recommended for use:

$$
20,25,30,35,40,45,50
$$

where the numbers indicate the specified characteristic compressive strength of the concrete at 28 days, in $\mathrm{N} / \mathrm{mm}^{2}$.

Use of a strength of $20 \mathrm{~N} / \mathrm{mm}^{2}$ is restricted to mass concretes.
In addition to its mechanical strength, the prescribed concrete shall fully comply with the durability requirements (minimum cement content and maximum water/cement ratio) that corresponds to the environment of the structural element, as outlined in 37.3.

The provisions and requirements in this Instruction are experimentally endorsed for strengths up to $50 \mathrm{~N} / \mathrm{mm}^{2}$; for any greater values, the appropriate adjustments shall be made.

## COMMENTS

The format included in the article is referred to concretes designed by properties. This corresponds to the more general and recommended procedure for the denomination of reay-mixed concretes, specially in edification. In the case of concretes designed by composition following format is recommended:
T - D -G/C/TM/A

Where $D$ means a concrete designed by composition and $G$ is the cement content, in $\mathrm{Kg} / \mathrm{m} 3$, prescribed by the client. Other parameters have the same meaning than before.

For concrete with a characteris Figure 37.2.4.a tic strength greater than $50 \mathrm{~N} / \mathrm{mm}^{2}$, this Instruction includes Annex 11 with recommendations for its use.

### 39.3 Characteristic stress-strain diagram for concrete.

The characteristic stress-strain diagram for concrete depends on numerous variables: age of the concrete, loading duration, shape and type of section, nature of stresses, type of aggregate and moisture conditions etc.

For practical purposes simplified characteristic diagrams may be used due to the difficulty presenting by having a concrete stress-strain diagram that is applicable to the particular case being studied,.

## COMMENTS

It may be considered that, from a qualitative point of view, the unitary concrete stress-strain diagrams adopt the following forms (Figures 39.3.a and 39.3.b):


RATE BETWEEN CONCRETE STRESS AND COMPRESSIVE STRENGTH IN CILINDERS

Figure 39.3.a


RATE BETWEEN CONCRETE STRSS AND
COMPRESSIVE STREGTH IN CILINDERS

Figure 39.3.b

### 39.4 Concrete design strength.

The design strength of concrete (under compression $f_{c d}$ or under tension $f_{c t, d}$ ) is the corresponding project characteristic strength $f_{c k}$ divided by a partial safety factor $\gamma_{c}$, which takes the values indicated in Article 15.

### 39.5 Design stress-strain diagram for concrete.

For the design of sections subject to normal stresses, one of the following diagrams should be adopted for Ultimate Limit States.
a) Parabolic-rectangular stress-strain diagram.

This is formed by a second-degree parabola and a straight line (Figure 39.5.a). The vertex of the parabola is found on the $x$-axis at $0.2 \%$ (yield strain of the concrete under compression) and the extreme vertex of the rectangle at $0.35 \%$ (yield strain of the concrete under flexure). The highest y-axis value on this diagram corresponds to a compression equal to $0.85 f_{c d}$, where $f_{c d}$ is the design compressive strength of the concrete.


Figure 39.5.a Parabolic-rectangular design diagram.
b) Rectangular diagram.

It consists of a rectangle with height $y$ which depends on the depth of the neutral axis $x$ in figure 39.5.b (for the usual case of $x \leq h y=0.8 x$ ) and with a width of $0.85 f_{c d}$.


Figure 39.5.b Rectangular design diagram.
c) Other design diagrams, such as parabolic, bilinear, trapezoidal, etc.

These are acceptable, always provided that the results they produce show satisfactory agreement with those of the parabolic-rectangular diagram or err on the side of safety.

### 39.6 Modulus of elasticity of concrete

For instantaneous or rapidly varying loads, the initial tangent modulus of elasticity (the slope of the tangent at the origin of the real curve $\sigma-\varepsilon$ ) at $j$ days may be taken as being equal to:

$$
E_{0 j}=10.000 \sqrt[3]{f_{c m, j}}
$$

In this expression, $f_{c m, j}$ is the mean compressive strength of the concrete at the age of $j$ days and must be expressed in $\mathrm{N} / \mathrm{mm}^{2}$ in order to obtain $E_{0 j}$ in $\mathrm{N} / \mathrm{mm}^{2}$.

The instantaneous secant modulus of elasticity $E_{j}$ (slope of the secant) will be:

$$
E_{j}=8.500 \sqrt[3]{f_{c m, j}}
$$

This expression is valid provided that the stresses in service do not exceed $0.45 f_{c ;}$, where $f_{c j}$ is the characteristic compressive strength of concrete at $j$ days.

## COMMENTS

$E_{0 j}$ and $E_{j}$ are average values of the modulus of elasticity and depend on the average strength of the concrete and not on the characteristic strengthWhenever the real average strength of the concrete is known, it is preferable to use it in order to calculate this modulus. In the opposite case, it is possible to estimate the average strength at 28 days, $f_{c m}$, from the characteristic strength at the same age, $f_{c k}$, using the expression $f_{c m}=f_{c k}+8$ $\mathrm{N} / \mathrm{mm}^{2}$, which is valid if the production conditions are acceptable (see the comments to Article $86^{\circ}$ ). From $f_{c m}$, Table 30.4.b, permits $f_{c m, j}$, to be estimated, and calcultate $E_{0 j}$ and $E_{j}$ by means of the expressions given in the article.

The modulus of elasticity of the concrete depends on the type of aggregate used to make the concrete. The expressions proposed in the article correspond to concrete produced with aggregates having average properties of the quartzite type. If it is necessary to obtain greater accuracy in the modulus value for the specific type of aggregate that is to be employed, and in the absence of real tests, then the modulus obtained in accordance with the article shall be multiplied by a coefficient $\alpha$, the value of which is given in Table 39.6.a.

Table 39.6.a Correction coefficient $\alpha$ for the modulus of elasticity in function of type of aggregate

| AGGREGATE |  | VALUE OF $\boldsymbol{\alpha}$ |
| :---: | :---: | :---: |
| QUARTZITE | 1 |  |
| LIMESTONE | SANDSTONE | 0.70 |
|  | NORMAL | 0.90 |
| ${ }^{(1)}$ OFITE, BASALT, <br> AND OTHER <br> VOLCANIC ROCKS | DENSE | 1.2 |
|  | POROUS | 0.9 |
| ${ }^{(2)}$ GRANITE AND OTHER DEEP-SEATED ROCKS |  | 1.2 |
| DIABASE |  | 1.1 |

(1) This group includes rocks such as riolite, dacite, andesite and ofite. The rocks present in this group (ofite, basalt and other volcanic rocks) normally have a low level of porosity and high density, but in some cases
they can have relatively high porosity, as shown, for example, by high absorption coefficients of $3.5 \%$ or higher. For this reason, the table provides a value of 1.2 for normal cases, together with a value of 0.9 for cases of high porosity.
(2)

This group includes rocks such as sienite and diorite.
In addition to the type of aggregate, there are other factors, such as the size of the aggregate, the type of cement, the water/cement ration or the stress level, which all influence the modulus of elasticity. In particular, the real modulus of elasticity may be noticeably less than the indicated values when concrete with high characteristic strength are used, with high initial strength and subject to strong levels of stress.

The expressions presented in the article relate the values at any age of compressive strength and modulus of elasticity of concrete. If a greater accuracy in the evaluation of the modulus of elasticity is required at ages other than at 28 days, it should be taken into account that the increase in the modulus with age is not equal to that undergone by the compressive strength. This evaluation may be performed with greater accuracy by multiplying the modulus value given by the expression in the Article for 28 days, by the parameter $\beta$, as indicated in Table 39.6.b.

Table 39.6.b Correction coefficient $\beta$ of the modulus of elasticity as a function of age

| Value of $\beta$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Concrete age (days) | $\mathbf{3}$ | $\mathbf{7}$ | $\mathbf{2 8}$ | $\mathbf{9 0}$ | $\mathbf{3 6 5}$ |
| Normal hardening <br> concretes (1) | 0.63 | 0.80 | 1.00 | 1.09 | 1.16 |
| Rapid hardening <br> concretes (1) | 0.74 | 0.87 | 1.00 | 1.07 | 1.09 |

(1) See the definition for rapid or normal hardening concrete in 30.3.

In those structures where strain and its control are especially important, whether due to magnitude, as in the case of very slender structures, or because of the forces and behaviour of the actual structure itself, as in the case of evolutionary or phased constructions, tests of the concretes to be used on site should be performed in order to obtain the most realistic results as possible of the modulus of elasticity.

### 39.7 Concrete shrinkage

When assessing shrinkage, the various variables that influence the phenomenon shall be taken into account, particularly: the degree of environmental humidity, the thickness or smallest dimension of the member, the concrete composition and the time elapsed since execution, which marks the duration of the phenomenon.

## COMMENTS

Shrinkage deformation of concrete may be evaluated by using the following expression:

$$
\varepsilon_{\mathrm{cs}}\left(\mathrm{t}, \mathrm{t}_{\mathrm{s}}\right)=\varepsilon_{\mathrm{cs} 0} \beta_{\mathrm{s}}\left(\mathrm{t}-\mathrm{t}_{\mathrm{s}}\right)
$$

where:

| $t$ | concrete age at the time of evaluation, in days. |
| :--- | :--- |
| $t_{s}$ | concrete age at the commencement of shrinkage, in days. |
| $\varepsilon_{c s 0}$ | basic coefficient of shrinkage. |

$$
\begin{gathered}
\varepsilon_{\mathrm{cs} 0}=\varepsilon_{\mathrm{s}} \beta_{\mathrm{HR}} \\
\varepsilon_{\mathrm{s}}=\left(570-5 \mathrm{f}_{\mathrm{ck}}\right) 10^{-6} \text { with } \mathrm{f}_{\mathrm{ck}} \text { in } \mathrm{N} / \mathrm{mm}^{2}
\end{gathered}
$$

For structures in air (HR<100\%):

$$
\beta_{H R}=-1,55\left(1-\left(\frac{H R}{100}\right)^{3}\right)
$$

For submerged structures:

$$
\beta_{H R}=0,25
$$

$H R \quad$ Relative humidity as a percentage.
e coefficient that defines the time development of the shrinkage.

$$
\beta_{s}\left(t-t_{s}\right)=\sqrt{\frac{t-t_{s}}{0,035 e^{2}+\left(t-t_{s}\right)}}
$$

Average thickness in millimetres.

$$
e=\frac{2 A_{c}}{u}
$$

$$
\begin{array}{ll}
A_{c} & \text { cross-section } \\
u & \text { The perimeter in contact with the atmosphere }
\end{array}
$$

For various values of the involved variables, the value of shrinkage deformation at various ages, taking the end of curing as the origin, in accordance with the proposed model and for normal weight concretes, may be taken from Table 39.7.

Table 39.7 Shrinkage values $\left[10^{-6}\right]$

| $\begin{gathered} \mathrm{t}-\mathrm{t}_{\mathrm{s}} \\ \text { [days] } \end{gathered}$ | Relative humidity [\%] |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 50 |  |  | 60 |  |  | 70 |  |  | 80 |  |  |
|  | Average thickness [mm] |  |  |  |  |  |  |  |  |  |  |  |
|  | 50 | 150 | 600 | 50 | 150 | 600 | 50 | 150 | 600 | 50 | 150 | 600 |
| 14 | -193 | -69 | -17 | -173 | -61 | -15 | -145 | -51 | -13 | -107 | -38 | -10 |
| 30 | -262 | -99 | -25 | -235 | -89 | -23 | -197 | -75 | -19 | -146 | -55 | -14 |
| 90 | -369 | -166 | -44 | -331 | -149 | -39 | -277 | -125 | -33 | -206 | -93 | -24 |
| 365 | -466 | -292 | -87 | -417 | -262 | -78 | -350 | -219 | -65 | -260 | -163 | -49 |
| 1825 | -507 | -434 | -185 | -454 | -388 | -165 | -381 | -326 | -139 | -283 | -242 | -103 |
| 10000 | -517 | -499 | -345 | -463 | -448 | -309 | -388 | -375 | -259 | -288 | -279 | -192 |

This model allows the use of correction coefficients in order to take the type of cement and curing temperature into account. The specialised literature should be consulted for this purpose.

### 39.8 Concrete creep

The stress-dependent strain at time $t$, for a constant stress $\sigma\left(t_{0}\right)$, less than $0.45 f_{c m}$, applied at $t_{0}$, may be estimated in accordance with the following criterion:

$$
\varepsilon_{c \sigma}\left(t, t_{0}\right)=\sigma\left(t_{0}\right)\left(\frac{1}{E_{0, t_{0}}}+\frac{\varphi\left(t, t_{0}\right)}{E_{0,28}}\right)
$$

where $t_{0}$ and $t$ are expressed in days.
The first expression in the brackets represents the instantaneous strain for unit stress, and the second represents the creep deformation, where:

| $E_{0,28}$ | Modulus of elasticity of concrete at 28 days, as defined in 39.6. |
| :--- | :--- |
| $E_{0, t 0}$ | Modulus of elasticity of concrete at the time of loading $\mathrm{t}_{0}$, defined in 39.6. |
| $\varphi\left(t, t_{0}\right)$ | Creep coefficient. |

## COMMENTS

The creep coefficient may be obtained by using the following formula:

$$
\varphi\left(t, t_{0}\right)=\varphi_{0} \beta_{c}\left(t-t_{0}\right)
$$

$\varphi_{0} \quad$ basic creep coefficient, as given by the expression:

$$
\begin{aligned}
& \varphi_{0}=\varphi_{\text {HR }} \beta\left(f_{c m}\right) \beta\left(t_{0}\right) \\
& \varphi_{\text {HR }}=1+\frac{100-\mathrm{HR}}{9,9 \mathrm{e}^{1 / 3}} \\
& \beta\left(f_{c m}\right)=\frac{16,8}{\sqrt{f_{c k}+8}} \\
& \beta\left(t_{0}\right)=\frac{1}{0,1+t_{0}^{0,2}}
\end{aligned}
$$

$\beta_{c}\left(t-t_{0}\right) \quad$ function that describes the development of creep with time.

$$
\begin{gathered}
\beta_{\mathrm{c}\left(\mathrm{t}-\mathrm{t}_{0}\right)}=\left[\frac{\left(\mathrm{t}-\mathrm{t}_{0}\right)}{\beta_{\mathrm{H}}+\left(\mathrm{t}-\mathrm{t}_{0}\right)}\right]^{0,3} \\
\beta_{\mathrm{H}}=1,5 \mathrm{e}\left\lfloor 1+(0,012 \mathrm{HR})^{18}\right\rfloor+250 \leq 1500
\end{gathered}
$$

In the previous expressions $e$ is the average thickness (see 39.7), in mm .
In order to obtain the delayed deformation having a concrete stress origin, the formulae in this section has an empirical base. The calibration is carried out from laboratory tests on concrete specimens subjected to compression.

This formula permits the use of correction coefficients in order to consider the influence of the following factors:

- Type of cement and curing temperature, which may be taken into account by modifying the age when the concrete was loaded.
- Tensions within the range of $0.45 f_{c m, t 0}<\left|\sigma_{\mathrm{c}}\right|<0.6 f_{c m, t 0}$. The non-linearity of the creep in this case is evaluated by multiplying the basic creep coefficient _o by an expression that depends on the ratio of applied stress/strength and the specialised literature should be consulted for this purpose.

For various values of the involved variables, the creep coefficient value at 10,000 days, in accordance with the proposed model, may be obtained from Table 39.8. In this table values correspond to a concrete with characteristic strength of $35 \mathrm{~N} / \mathrm{mm}^{2}$

Table 39.8 Creep coefficient values

| Age when loaded $t_{0}$ [days] | Relative humidity [\%] |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 50 |  |  | 60 |  |  | 70 |  |  | 80 |  |  |
|  | Average thickness [mm] |  |  |  |  |  |  |  |  |  |  |  |
|  | 50 | 150 | 600 | 50 | 150 | 600 | 50 | 150 | 600 | 50 | 150 | 600 |
| 1 | 5.4 | 4.4 | 3.6 | 4.8 | 4.0 | 3.3 | 4.1 | 3.6 | 3.0 | 3.5 | 3.1 | 2.7 |
| 7 | 3.8 | 3.1 | 2.5 | 3.3 | 2.8 | 2.3 | 2.9 | 2.5 | 2.1 | 2.5 | 2.2 | 1.9 |
| 14 | 3.3 | 2.7 | 2.2 | 2.9 | 2.4 | 2.0 | 2.5 | 2.2 | 1.8 | 2.2 | 1.9 | 1.7 |
| 28 | 2.9 | 2.4 | 1.9 | 2.6 | 2.1 | 1.8 | 2.2 | 1.9 | 1.6 | 1.9 | 1.7 | 1.5 |
| 60 | 2.5 | 2.1 | 1.6 | 2.2 | 1.9 | 1.5 | 1.9 | 1.7 | 1.4 | 1.6 | 1.4 | 1.3 |
| 90 | 2.3 | 1.9 | 1.5 | 2.0 | 1.7 | 1.4 | 1.8 | 1.5 | 1.3 | 1.5 | 1.3 | 1.2 |
| 365 | 1.8 | 1.4 | 1.2 | 1.6 | 1.3 | 1.1 | 1.4 | 1.2 | 1.0 | 1.2 | 1.0 | 0.9 |
| 1800 | 1.3 | 1.1 | 0.8 | 1.1 | 1.0 | 0.8 | 1.0 | 0.9 | 0.7 | 0.8 | 0.7 | 0.7 |

### 39.9 Poisson's ratio.

For Poisson's ratio for elastic deformations under normal service stresses, a mean value equal to 0.20 should be used

### 39.10 Coefficient of thermal expansion

The coefficient of thermal expansion of concrete is taken as equal to $10-5$

## COMMENTS

Tests have shown that this coefficient may vary in a relatively high proportion (in the order of $\pm 30$ percent). This coefficient depends on the type of cement, type of aggregates, the concrete composition, the hygrometry and the section dimensions.

As far as the aggregates are concerned, the lowest values are obtained with limestone aggregates and the highest with silica aggregates.

