# Serviceability Limit State Design 

## CHAPTER XI

## Article 49 Cracking Limit State

### 49.1 General considerations

For verifications with regard to the Cracking Limit State, the effects of the action consist of the internal stresses in the sections ( $\sigma$ ) and the crack widths ( $w$ ) that they cause, where applicable.

In general, both $\sigma$ and $w$ are obtained from the design actions and the combinations indicated in Chapter III for Serviceability Limit States.

The stresses should be calculated from the actions in accordance with that explained in Chapter V. The internal stresses, crack widths or other verification criteria should be evaluated in accordance with the specifications of 49.2 in the case of perpendicular stresses, 49.3 for shear stresses, and 49.4 for torsional stresses.

## COMMENTS

The appearance of cracking is usually inevitable in concrete structure, but this does not necessarily represent any obstacle to the normal use of the same, provided that the maximum widths are always limited to values that are compatible with the requirements for durability, functionality, watertight properties and appearance.

The cracking dealt with in this article corresponds to that produced due to direct actions or imposed deformations.

Cracking due to plastic shrinkage or settling, which occurs during the first hours after mixing, is not covered by this article and its control should by defined by suitable composition of the concrete mix and manufacture of the concrete, together with its pouring and curing.

Neither does this article take into account any cracking that is caused by expansive chemical reactions that take place in the hardened concrete,

The evaluation of the behaviour of the section for the verification of this limit state may be carried out by considering a linear elastic behaviour of the compressed concrete and of the reinforcement and ignoring the tensile strength capacity of the concrete.

The geometric characteristics of the section under consideration should be representative of its state at the time of verification (18.2.3), especially taking into account the prestressing steel (pre-tensioned, posttensioned, bonded or not), together with the state of the cracking.

### 49.2 Cracking due to perpendicular stresses

### 49.2.1 The appearance of cracks due to compression

Under the combination of most unfavourable actions corresponding to the phase under analysis, the compressive stresses in the concrete should satisfy the condition:

$$
\sigma_{c} \leq 0,60 f_{c k, j}
$$

where:
$\sigma_{c} \quad$ Compressive stress in the concrete in the verification situation.
$f_{c k, j} \quad$ Value assumed in the project for the compressive characteristic strength at $j$ days (the age of the concrete at the phase under consideration)

### 49.2.2 Decompression Limit State

Design in relation to the Decompression Limit State consists of verifying that, under the combination of actions corresponding to the phase being studied, the concrete is not decompressed in any fibre of the section.

### 49.2.3 Cracking due to tension. Verification criteria

General verification of the Limit State of Cracking due to tension consists of satisfying the following inequality

$$
w_{k} \leq w_{\max }
$$

where:
$w_{k} \quad$ Characteristic crack width.
$w_{\text {máx }} \quad$ Maximum crack width as defined in 49.2.4.

### 49.2.4 Maximum values for crack width

In the absence of any specific requirements for reinforced concrete elements (watertightness, etc.) and under the combination of quasi-permanent actions, the maximum crack widths for the various environments as defined in table 8.2.2 are given in table 49.2.4.

In the absence of any specific requirements for prestressed concrete elements, and under the combination of frequent actions, the maximum crack widths for the various environments as defined in table 8.2.2 are given in table 49.2.4.

Table 49.2.4

| Exposure class | $\mathbf{w}_{\text {max }}[\mathrm{mm}]$ |  |
| :---: | :---: | :---: |
|  | Reinforced <br> concrete | Prestressed <br> concrete |
| I | 0.4 | 0.2 |
| $\mathrm{IIa}, \mathrm{Ilb}, \mathrm{H}$ | 0.3 | $0.2^{1)}$ |
| $\mathrm{IIIa}, \mathrm{IIIb}, \mathrm{IV}, \mathrm{F}$ | 0.2 | Decompression |
| $\mathrm{IIIc}, \mathrm{Qa}, \mathrm{Qb}, \mathrm{Qc}$ | 0.1 |  |

[^0]
### 49.2.5 General method for calculating crack width

The characteristic crack width should be calculated using the following expression:

$$
w_{k}=\beta_{S_{m}} \varepsilon_{s m}
$$

where:
$\beta \quad$ is the coefficient that relates the average crack width to the characteristic value, taking the value 1.3 for cracking produced by indirect actions only and 1.7 for all other cases.
$s_{m} \quad$ is the average crack separation, expressed in mm .

$$
s_{m}=2 c+0,2 s+0,4 k_{l} \frac{\phi A_{c, e f i c a z}}{A_{s}}
$$

$\varepsilon_{s m} \quad$ Mean elongation of the reinforcement, taking into account the collaboration of the concrete between the cracks.

$$
\varepsilon_{s m}=\frac{\sigma_{s}}{E_{s}}\left[1-k_{2}\left(\frac{\sigma_{s r}}{\sigma_{s}}\right)^{2}\right]-0,4 \frac{\sigma_{s}}{E_{s}}
$$

c The concrete covering.
$s \quad$ The distance between cross-sectional bars. If $s>15 \varnothing, s=15 \varnothing$ is taken.
In the case of reinforced beams with $n$ bars, $s=b / n$ is taken, where $b$ is the width of the beam.
$k_{1} \quad$ is the coefficient representing the influence of the tension diagram in the section, having a value of

$$
k_{l}=\frac{\varepsilon_{1}+\varepsilon_{2}}{8 \varepsilon_{1}}
$$

where $\varepsilon_{1} \mathrm{y} \varepsilon_{2}$ are the greater and lesser deformations calculated in a cracked section at the extremes of the tension zone (figure 49.2.5.a).

Figure 49.2.5.a

$\varnothing \quad$ The diameter of the thickest bar under tension, or equivalent diameter in the case of a bundle of bars.
$A_{c, \text { eficaz }}$ is the area of concrete in the covering zone, as defined in figure 49.2.5.b, where the tension bars effectively influence the crack width.

Figure 49.2.5.b.

CASE 1

CASE 2
BEAMS WITH....


CASO 2
VIGAS CON $\mathrm{s} \leqslant 15 \varnothing$


CASO 3
VIGAS PLANAS, MUROS, LOSAS CON s $>15 \phi$

CASE 3
PLANE BEAMS, WALLS, SLABS WITH...
$A_{s} \quad$ Total cross-section of the reinforcement located in the area $A_{c, \text { e eficaz }}$.
$\sigma_{s} \quad$ Service stress in the reinforcing steel under the assumption that the section is cracked.
$E_{s} \quad$ Modulus of longitudinal deformation in the reinforcing steel.
$k_{2} \quad$ Coefficient having a value of 1.0 in cases of instantaneous, non-repeated load and 0.5 in all others.
$\sigma_{s r} \quad$ Stress in the reinforcement in the cracked section at the moment the concrete cracks, which is assumed to occur when the tensile stress in the fibre under greatest tension in the concrete reaches the value $f_{c t, m}$ (39.1).

For a prestressed section with bonded prestressing steel and reinforcing steel, the crack width is calculated as if the section were a reinforced concrete section, taking the prestressing action into account as an external action and the reinforcing steel as existing in the section.

As an alternative, in prestressed sections with bonded prestressing steel and no reinforcing steel, if the increase in stress in the active reinforcement steel due to the action of external loads is less than $200 \mathrm{~N} / \mathrm{mm}^{2}$, then, as a simplification, it may be assumed that a crack width of greater than 0.2 mm has not been reached.

COMMENTS

The expressions given in the article enable the average crack opening to be calculated from its average separation $s_{m}$ and the average steel deformation $\varepsilon_{s m}$. The experimental results show that the dispersion is greater for cracking caused by direct actions than for cracking caused by indirect actions. For this reason, the coefficient $\beta$ which enables the characteristic crack width to be obtained from the average widths is 1.3 for cracking caused by indirect actions only and 1.7 in all other cases.

For the calculation of the stresses in reinforcement under tension ( $\sigma_{\mathrm{s}}$ and $\sigma_{\mathrm{sr}}$ ), in reinforced concrete elements subjected to pure bending, the general expressions defined in Appendix 9 may be employed. In a simplified fashion, these stresses may be evaluated with the following expressions:

$$
\begin{gathered}
\sigma_{s r}=\frac{M_{f i s}}{0,8 d A_{s}} \\
\sigma_{s}=\frac{M_{k}}{0,8 d A_{s}}
\end{gathered}
$$

where:
$M_{\text {fis }} \quad$ The moment when the fibre under greatest tension reaches a value of $f_{c t, m}$.
$M_{k} \quad$ Moment at which the Cracking Limit State verification is made.

### 49.3 Limitation of cracking due to shear stress

It may be assumed that cracking due to shear stress is suitably controlled if the spacing between stirrups, as defined in table 49.3, is met.

Table 49.3
The spacing of beam stirrups for the control of cracking

| $\frac{V_{r d}-3 V_{c u}}{A_{\alpha} d} \operatorname{sen} \alpha\left[\frac{N}{{m m^{2}}^{2}}\left(^{*}\right)\right.$ | Stirrup spacing (mm) |
| :---: | :---: |
| $<50$ | 300 |
| 75 | 200 |
| 100 | 150 |
| 150 | 100 |
| 200 | 50 |

(*) $^{*} \quad$ The meaning of all the variables is the same as those of Article 44.
No verification is required for those members where no shear reinforcement is needed.

### 49.4 Limitation of cracking due to torsion

It can be assumed that cracking due to torsional stress is suitably controlled if the spacing of transverse reinforcement is within the following limitations:

$$
\begin{gathered}
s_{t} \leq \frac{a}{2} \\
s_{t} \leq \frac{b}{3} \\
s_{t} \leq 200 \mathrm{~mm}
\end{gathered}
$$

where:
a is the smallest cross-sectional dimension of the member.
$b \quad$ is the largest cross-sectional dimension of the member.

## Article 50 Deformation Limit State

### 50.1 General considerations

The Deformation Limit State will be met if the movements (whether deflections or rotations) in the structure or structural element are smaller than specific maximum limit values.

The verification of the Deformation Limit State should be carried out in those cases where the deformations could lead to the construction being taken out of service for functional, aesthetic or any other reasons.

The study of the deformations should be carried out for the service conditions corresponding to the problem under consideration, in accordance with the combination of criteria given in 13.3.

The total deformation produced in a concrete element is the sum of the various partial deformations that are produced over time as a result of the applied loads, creep and shrinkage of the concrete, and the relaxation of the prestressing steel.

## COMMENTS

The deformation of the element is a function of the materials' properties, of the actions, of the geometry, reinforcement and the connections of the element. All this means that the estimation of the deformations is complex and that they should be considered as a random variable that may only evaluated approximately.

In the case of members that support non-structural elements, the project designer should consider that the need to avoid any damage to such elements could place more limitations, as far as structural deformation is concerned, than what would be required for a structure that is considered in isolation, as in the case of partitions and enclosures that rest on floor slabs and concrete beams.

A distinction should be made between:

- Total deflection due to the total acting load. This consists of the instantaneous deflection produced by all the loads plus the long-term time-dependent deflection due to the permanent and quasi-permanent loads as from their action.
- The active deflection with respect to a damageable element produced from the moment when this element is constructed. Its value is therefore equal to the total deflection less that which has already been produced up to the moment when the element is constructed.

The maximum acceptable values for the deflections depend on the type and function of the structure, of the functional conditions it has to satisfy and on the conditions that might be imposed by other non-structural elements that it supports. For all these reasons, it is difficult the establish any general limiting values, and therefore, these should be defined for each case in accordance with the corresponding specific characteristics.

In general, for normal buildings, in the absence of more precise requirements resulting from specific conditions, the limiting value of $L / 250$ may be established for the total deflection in terms relative to the length $L$ of the element that is being verified.

In order to prevent cracking in partition walls, in the absence of more precise criteria for each specific case, the limiting value for active deflection may be defined in relative terms as the length of the element that is undergoing verification, as $L / 400$. In all cases, existing data in the literature, which has been obtained from real cases of damages, indicates that the active deflection should not exceed 1.0 cm if partition cracking problems are to be prevented.

In extreme cases, the designer may require that a constructive process be carried out in order to minimise this active deflection that usually affects partition wall cracking.

### 50.2 Elements subjected to pure or combined bending stress

### 50.2.1 General Method

The most general method for calculating deflection consists of a structural step-by-step time analysis, in accordance with the criteria given in Article 25, where, for each instant of time, the deformation is calculated by double integration of the curvatures along the length of the member.

## COMMENTS

General analysis is complex, and its use is usually only justified in very special situations, where the deformation controls require extreme precision.

### 50.2.2 Simplified method

This method may be applied to reinforced concrete beams and flat slabs. The deflection is considered to consist of the sum of an instantaneous deflection and a timedependent deflection that is due to quasi-permanent loads.

### 50.2.2.1 Minimum depths

The verification of deflection will not be necessary when the span/effective depth ratio of the element under consideration is equal to, or less than, the values given in table 50.2.2.1.

Table 50.2.2.1: $L / d$ ratios in reinforced concrete structural elements subjected to pure bending

| STRUCTURAL SYSTEM | Heavily reinforced <br> elements <br> $\left(\rho=A_{s} / b_{o} d=0.012\right)$ | Slightly <br> reinforced <br> elements |
| :---: | :---: | :---: |
| $\left(\rho=A_{s} / b_{0} d=0.004\right)$ |  |  |$|$

1 An end is considered as being continuous if the corresponding moment is equal to, or greater than, $85 \%$ of the perfectly restrained moment.
2 In one-way slabs, the given slenderness values refer to the smaller span.
3 In flat slabs on point supports (columns), the given slenderness values refer to the larger span.
table 50.2.2.1 corresponds to normal situations in building and for elements reinforced with steel $f_{y k}=500 \mathrm{~N} / \mathrm{mm}^{2}$.

## COMMENTS

The stiffness of a member subjected to bending depends to a large extent on its depth. In general, therefore, placing pre-established upper limits on the values of the span/effective depth ratio (L/d) of this type of member could guarantee that its deformation is maintained within tolerable margins.

In general in table 50.2.2.1, beams are considered as being heavily reinforced ( $\rho=A_{s} / b_{0} d=0.012$ ), whereas slabs may be considered as slightly reinforced elements ( $\rho=A_{s} / b_{0} d=0.04$ ). The tension reinforcement ratios given in the table are the ones strictly necessary for equilibrium in ULS, not the actual ones, which are usually greater. When the strictly necessary tension reinforcement ratio in the critical section of the member (central section in bays or cantilever springing) is known, then linear interpolation may be performed between the values given in the table, applying any corresponding applicable correction factors to the resulting value.

### 50.2.2.2 Calculation of instantaneous deflection

In order to calculate the instantaneous deflections in cracked, constant-section elements in the absence of more rigorous methods, the following simplified method may be used:

1. The equivalent second moment of area of a section is defined as the value $I_{e}$ given by:

$$
I_{e}=\left(\frac{M_{f}}{M_{a}}\right)^{3} I_{b}+\left[1-\left(\frac{M_{f}}{M_{a}}\right)^{3}\right] I_{f}-I_{b}
$$

where:
$M_{a} \quad$ The total bending moment applied to the section up to the instant when the deflection is evaluated.
$M_{f} \quad$ Nominal cracking moment of the section, which is calculated using the following expression:

$$
M_{f}=f_{c t, f l} W_{b}
$$

$f_{c t, f} \quad$ Flexural tensile strength of the concrete, which for the sake of simplicity may be taken as being equal to $0.37 f_{c k, j}^{2 / 3}$ for $f_{c t, t t}$ and $f_{c k, j}$ in $\mathrm{N} / \mathrm{mm}^{2}$.
$W_{b} \quad$ Section modulus of the gross section with respect to the fibre under greatest tension.
$I_{b} \quad$ The moment of inertia of the gross section.
$I_{f} \quad$ Second moment of area of the cracked section in pure bending, which is obtained by ignoring the zone of concrete under tension and making the areas of prestressing and reinforcing steel uniform by multiplying them by the coefficient of equivalence.
2. The maximum deflection of an element may be obtained from the Strength of Materials formulae, adopting the modulus defined in 39.6 as the modulus of longitudinal strain of concrete, and as the constant second moment of area for the entire member the second moment of area for the reference section defined as follows:
a) In simply supported elements or continuous elements, the mid-span section.
b) The support section in cantilever elements.

## COMMENTS

The cracked stiffness value for the rectangular and " $T$ " sections or those that may be assimilated as such, subjected to pure bending may be found in Appendix 9.

### 50.2.2.3 Calculation of time-dependent deflection

The additional time-dependent defection, produced by long-duration loads, resulting from creep and shrinkage deformation, may be estimated, except where greater precision is justified, by multiplying the instantaneous deflection value by the corresponding factor $\lambda$

$$
\lambda=\frac{\xi}{1+50 \rho^{\prime}}
$$

where:
$\rho^{\prime} \quad$ Compression reinforcement ratio $A^{\prime}$ referred to the area of the effective section $b_{0} d$ in the referenced section.

$$
\rho^{\prime}=A \frac{\prime}{b_{0} d}
$$

$\xi \quad$ is the coefficient function of load duration, as taken from the list of values given below:

| 5 or more years | 2.0 |
| :--- | :--- |
| 1 year | 1.4 |
| 6 months | 1.2 |
| 3 months | 1.0 |
| 1 month | 0.7 |
| 2 weeks | 0.5 |

For age $j$ of the load and $t$ for the deflection calculation, the value of $\xi$ to be taken into account for the calculation of $\lambda$ is $\xi(t)-\xi(j)$.

In the situation where the load is applied by fractions $P_{1}, P_{2} \ldots, P_{n}$, the value of $\xi$ given by the following may be adopted:

$$
\xi=\left(\xi_{1} P_{1}+\xi_{2} P_{2}+\ldots+\xi_{n} P_{n}\right) /\left(P_{1}+P_{2}+\ldots+P_{n}\right)
$$

### 50.3 Elements subjected to torsional stress

The rotation of linear members or elements that are subjected to torsion may be obtained by simple integration of the rotations per unit length deriving from the following expression:

$$
\begin{aligned}
\theta & =\frac{T}{0,3 E_{c} I_{j}} \\
\theta & =\frac{T}{0,1 E_{c} I_{j}}
\end{aligned} \quad \text { for non-cracked sections }
$$

where:
$T \quad$ Torsional moment in service.
$E_{c} \quad$ Modulus of elasticity of concrete as defined in 39.6.
$I_{j} \quad$ Torsional second moment of area of the gross concrete section.

### 50.4 Elements subjected to pure tension

The deformation in elements subjected to pure tension may be calculated by multiplying the average unit elongation of the reinforcement $\varepsilon_{\text {sm }}$, obtained in accordance with 49.2.5, by the length of the element.

## Article 51 Vibration Limit State

### 51.1 General considerations

Vibrations may affect the behaviour of structures in service due to functional reasons. Vibrations may cause discomfort to the occupants or users or they may affect the functioning of equipment that is sensitive to this kind of phenomenon, etc.

## COMMENTS

Vibrations in structures may be caused by various actions, such as:

- Rhythmic movement produced by people walking, running, jumping or dancing.
- Machinery.
- $\quad$ Gusts of winds or waves.
- Overload of highway or railway traffic.
- $\quad$ Certain construction procedures, such as pile-driving or sheet-piling, mechanical compacting of the ground etc.

Any vibrations that could lead to the structure collapsing, large deformations caused by resonance or the loss of strength due to fatigue, should all be taken into consideration during the verification of the structure's ultimate limit states.

### 51.2 Dynamic behaviour

Generally, in order to satisfy the vibrations limit state, the structure should be designed so that any natural frequencies of vibration are sufficiently distant from certain critical values.

## COMMENTS

The dynamic behaviour of concrete structures is difficult to classify in a precise manner, since it is influenced by the change in natural frequencies and by changes in the structure's rigidity conditions due to cracking or damping parameters. There is also the additional difficulty of classifying dynamic loads.

In the absence of more precise data or other criteria that may suggest other specific standards, table 51.2 provides the requirements that should be satisfied in those structures that are liable to undergo vibrations due to the rhythmic movement of people.

Table 51.2

| Structure | Frequency $(\mathrm{Hz})$ |
| :--- | :---: |
| Gymnasiums or sports pavilions | $>8.0$ |
| Dance or concert halls without fixed seating. | $>7.0$ |
| Dance or concert halls with fixed seating. | $>3.4$ |

In the case of pedestrian footbridges, structures with frequencies between 1.6 and 2.4 Hz and between 3.5 and 4.5 Hz should be avoided.


[^0]:    1) 

    It should also be verified that the prestressing tendon lies within the compression zone of the section.

